

## Color evolution of $2 \rightarrow 3$ processes

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**ABSTRACT:** The color structure needed for resummation of all colored  $2 \rightarrow 3$  processes is calculated using multiplet inspired  $s$ -channel bases. In this way the resulting matrices, describing the color structure, are guaranteed to obey simplifying symmetries.

**KEYWORDS:** QCD, Jets, NLO Computations.

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## 1. Introduction

In perturbation theory resummation becomes necessary when large logarithms compensate the smallness of the coupling constant and invalidate a fixed order calculation.

Large logarithms occur in the collinear region, where a large rapidity logarithm gives an effectively large phase space for radiation. In this case the radiation is coherent and acts as if it was emitted by a single parton. Large logarithms may also occur for wide angle radiation of soft gluons. For such radiation, where real and virtual gluon emissions cancel (at least for global observables, where there is no issue with out of the gap region gluons being prevented from radiating into the gap), the complication of the color structure can

be expressed in terms of a matrix containing color and phase space information: the soft anomalous dimension matrix [1–4]. This matrix has contributions both from the region of phase space where the exchanged gluons go on shell, sometimes referred to as the eikonal gluon contribution, and the region of phase space where the hard partons go on shell at intermediate steps, the Coulomb gluon region. The latter would in an Abelian theory give just an unobservable phase, but leads to a physically relevant change for a non-Abelian theory.

In principle, soft gluon resummation of this type is of relevance for any colored emission where there is a large logarithm in (hard scale/soft scale). However, largely due to the complicated color structure, the soft anomalous dimension matrix has so far only been calculated for the  $2 \rightarrow 2$  processes [1–7] and one  $2 \rightarrow 3$  process [8].

What is used in event generators is instead a color diagonal, leading  $N$ , approximation. This may work well for most inclusive observables, but is destined to break down under certain circumstances. Perhaps the simplest example of this type of observable is an observable with a rapidity gap, free from radiation above some veto transverse momentum  $Q_0$ , such that  $\log(\text{hard scale}/Q_0)$  is sufficiently large [9]. In this case color mixing effects are significant. In particular a colored exchange may change to an overall color singlet exchange by virtual gluon exchange, leading to a decreased radiation probability and an increased gap survival probability [10, 11]. The results for the gap-survival probability within a rapidity region  $Y$  have also been experimentally tested using events with rapidity gaps at HERA and the Tevatron [12–16, 10].

To go beyond events with 2 jets, clearly the color evolution for  $2 \rightarrow n$  matrix elements is needed for a  $n$ -jet event. However, due to the non-global nature of most observables, corrections from the color structure corresponding to  $2 \rightarrow n$  scattering, for all  $n$ , are in principle present even for two jet events with a rapidity gap [17–23]. To deal with this problem in its full complexity would probably require significant progress in our understanding of the group theory aspects of QCD. What is presented here are the results needed to calculate the soft anomalous dimension matrix for all  $2 \rightarrow 3$  processes, completing the work initiated in [8]. This would be the full story for a global observable with a large logarithm in (hard scale)/(veto scale) and a moderate  $Y$ . However, since most observables of interest are non-global, these results can alternatively be used to obtain the lowest order non-global correction for two jet events with a rapidity gap.

Apart from the aforementioned applications, this color structure information is also needed to investigate the effects of the so called super-leading logarithms, carrying an extra power of  $\log(\text{hard scale}/\text{veto scale})$  [24], and to see if the possible breakdown of factorization suggested in [25] for  $qq \rightarrow qqg$  persists to more complicated color states (as is anticipated).

The outline of this paper is as follows: First, to set the scene, gluon resummation is discussed in general in section 2, and exemplified with several  $2 \rightarrow 2$  processes. Then, in section 3 the construction of multiplet inspired bases is presented for  $2 \rightarrow 3$  processes. The resulting soft anomalous dimension matrices are discussed in section 4, and the explicit formulae are stated in appendix B. Finally some concluding remarks are made in section 5.

## 2. Gluon resummation

To perform the resummation of virtual gluon exchange on a hard scattering matrix element  $M$ , the effect of virtual gluon exchange on  $M$  has to be considered. This is perhaps most easily illustrated by a simple example. Consider therefore a hard scattering of two quarks  $q_a q_b \rightarrow q_c q_d$ . The *exchange* between the quarks  $q_a$  and  $q_b$  can form a total color singlet (from for example the exchange of an electro-weak boson) or a total color octet (as from exchange of a gluon). This can be described by the color tensors

$$\begin{aligned} C_{abcd}^1 &= \delta_{ac}\delta_{bd}, \\ C_{abcd}^8 &= t_{ca}^g t_{db}^g = \frac{1}{2} \left( \delta_{ad}\delta_{bc} - \frac{1}{N} \delta_{ac}\delta_{bd} \right). \end{aligned} \tag{2.1}$$

Similarly, the color structure corresponding to anti-quarks and gluons in the initial and final states, or more than 2+2 partons can be described. For this purpose, in general, delta functions in quark indices, delta functions in gluon indices, generators  $t_{bc}^a$ , and the symmetric and antisymmetric structure constants  $d_{abc}$  and  $f_{abc}$ ,

$$(if/d)_{abc} = 2(\text{Tr}[t^a t^b t^c](-/+)\text{Tr}[t^b t^a t^c]). \tag{2.2}$$

may be used.

The color states used for describing a colored amplitude form a vector space. Using the scalar product

$$\sum_{a,b,c,d\dots} C_{abcd\dots}^i (C^j)_{abcd\dots}^*, \tag{2.3}$$

where  $C^i$  denotes a certain color structure and  $a\dots d\dots$  are quark, anti-quark and gluon indices, a complete orthogonal set of color states can be found. The effect of exchanging a virtual gluon on a color state is to map the color vector into a linear combination of color states. This linear transformation can thus be described by a matrix.

### 2.1 2 → 2 processes

For a non-trivial color structure to arise there must be at least four colored particles [6]. In order to set the scene for the more complicated processes, and shed some light on the construction of a basis, some 2 → 2 processes are discussed below.

### 2.2 A simple example, qq → qq

Here a simple non-trivial case is considered, namely  $q_a q_b \rightarrow q_c q_d$  [3]. For the resummation to be successful (even in the Abelian case) we need to assume that only gluons strongly ordered in transverse momenta contribute. This assumption simplifies both the kinematics, as the softer momenta can be ignored, and the color algebra, as no nested color structures have to be considered. To deal with resummation of soft gluons, the exchange of *any* number of virtual gluons has to be considered. The first step is naturally to consider the exchange of only one extra gluon, attached in all possible ways to the hard scattering. For

example, a virtual gluon could be exchanged in the  $t$ -channel. This would change an initial  $t$ -channel color singlet exchange into a color octet exchange. An initial color octet exchange would transform into a combination of a color singlet exchange and a color octet exchange. The exchange of a  $u$ -channel gluon would have a similar effect, whereas a singlet would stay a singlet, and an octet would stay an octet, under a vertex correction exchange. The color structure further simplifies, as the color effect of an exchange between quark  $a$  and quark  $b$  is the same as the effect of an exchange between  $c$  and  $d$ , and similarly for  $ac$  and  $bd$  or  $ad$  and  $bc$ . The kinematic factors from phase space integration (azimuth and rapidity) over the regions in which radiation is forbidden can thus be summed as  $T = \Omega_{ab} + \Omega_{cd}$ ,  $U = \Omega_{ad} + \Omega_{bc}$  and  $V = \Omega_{ac} + \Omega_{bd}$ . All this information is described by the soft anomalous dimension matrix,

$$\mathbf{\Gamma}_{qq \rightarrow qq} = \begin{pmatrix} \frac{(N^2-1)V}{2N} & \frac{(N^2-1)(T+U)}{4N^2} \\ T+U & -\frac{UN^2+2T+2U+V}{2N} \end{pmatrix}, \quad (2.4)$$

where the phase space integrals over the azimuth and rapidity of the exchanged  $k'$  are

$$\Omega_{ij} = \frac{1}{2} s_{ij} \left[ \int_{\Omega} \frac{dy' d\phi'}{2\pi} \frac{k'_{\perp}{}^2 p_i \cdot p_j}{2p_i \cdot k' k' \cdot p_j} - \frac{1}{2} (1 - s_{ij}) i\pi \right] \quad (2.5)$$

in the relevant eikonal limit. Here  $s_{ij} = -1$  if the quarks (partons in general)  $i$  and  $j$  are both incoming or both outgoing, and  $s_{ij} = 1$  otherwise. (For each involved anti-quark and each outgoing gluon — with the present triple gluon convention, see eq. (2.12) — there is an additional overall minus sign.) These integrals have been evaluated for a topology in which radiation is forbidden within a rapidity region  $-Y/2 < y' < Y/2$  in for example [8]. With the above simplification all virtual gluons can be resummed as

$$\mathbf{M} = \exp \left( -\frac{2}{\pi} \int_{Q_0}^Q \alpha_s(k'_{\perp}) \frac{dk'_{\perp}}{k'_{\perp}} \mathbf{\Gamma} \right) \mathbf{M}_0, \quad (2.6)$$

where  $\mathbf{M}_0$  denotes the hard matrix element (as a vector in color space).

The  $qq \rightarrow qq$  basis eq. (2.1) is orthogonal but not normalized. To account for this, when calculating the cross section the scalar product of the color vectors has to be used,  $\sigma = \mathbf{M}^\dagger \mathbf{S}_{qq \rightarrow qq} \mathbf{M}$  with

$$\mathbf{S}_{qq \rightarrow qq} = \begin{pmatrix} N^2 & 0 \\ 0 & \frac{1}{4} (N^2 - 1) \end{pmatrix}. \quad (2.7)$$

As this is obtained by summing (rather than averaging) over incoming quarks a quark color averaging is expected for the hard scattering matrix. Similar normalization matrices will be needed for the other bases under consideration here.

As for the color algebra part, it should be noted that the  $qq \rightarrow qq$  scattering, here described by considering the exchange in  $t$ -channel, corresponds precisely to the  $q\bar{q} \rightarrow q\bar{q}$  scattering, but viewed in the  $s$ -channel. The result, eq. (2.4) can be used for any amplitude with four external quarks or anti-quarks. In principle, when the color algebra has been calculated in one case, it can be used also for the other. For this reason, to describe all

possible four-parton colored scattering processes, only three different color topologies are needed. The two remaining are briefly described below. Identical arguments reduce the number of different subprocesses that need to be considered for  $2 \rightarrow 3$  processes to three: four external quarks or anti-quarks and one gluon, two external quarks or anti-quarks and three gluons, or five external gluons.

Generally speaking a certain basis may be physically more illuminating. For example, using the  $t$ -channel basis as above, one can easily make a comparison to classical radiation from an accelerated (color) charge. The classical radiation is expected to be large when the acceleration is large. In the case of acceleration of colors, this corresponds to a  $t$ -channel octet (gluon) exchange, and indeed, one finds that the amount of radiation is significantly higher for a color octet exchange than for a color singlet exchange [11].

### 2.3 Color structure of $qg \rightarrow qg$

Colorwise, one may choose to describe this process in the  $s$ -channel basis, stating that a quark (3) and a gluon (8) can be in three different states  $8 \otimes 3 = 3 \oplus \bar{6} \oplus 15$ . The final quark and gluon must then be in the same state. One can thus write down 3 color tensors corresponding to  $3 \rightarrow 3$ ,  $\bar{6} \rightarrow \bar{6}$  and  $15 \rightarrow 15$ .

Alternatively, one can consider the exchange in the  $t$ -channel basis and conclude that the incoming quark and outgoing quark, counting as an incoming anti-quark, can be in a color singlet or a color octet states. The gluons must then also be in an octet or a singlet state respectively, but  $8 \otimes 8$  contains two octet to choose from. These octets may, without loss of generality, be divided into one piece which is symmetric in the incoming gluons and one piece which is antisymmetric. Alternatively one can thus use the basis corresponding to  $1 \rightarrow 1$ ,  $8 \rightarrow 8^s$  and  $8 \rightarrow 8^a$ .

A basis used for  $qg \rightarrow qg$  can clearly also be used to describe  $gg \rightarrow q\bar{q}$ ,  $\bar{q}q \rightarrow gg$  and  $\bar{q}g \rightarrow \bar{q}g$ .

### 2.4 Color structure of $gg \rightarrow gg$

Writing down the possible multiplets of two gluons gives, in SU(3)-inspired notation:

$$8 \otimes 8 = 1 \oplus 8 \oplus 8 \oplus 10 \oplus \bar{10} \oplus 27 \oplus 0. \tag{2.8}$$

Clearly the number of states are different from 8, 10 etc when  $N \neq 3$ , but the above notation will nevertheless be used throughout this paper. For three colors, the last state has multiplicity 0, and need thus not be considered. (For  $N = 2$  there are only three states on the right hand side of eq. (2.8) “1”, “8” and “27”, with multiplicity 1, 3 and 5 respectively.) The incoming gluons may be in any of the above multiplets and the outgoing must be in the same. This enables the construction of 9 different color states. A basis corresponding to the 9 possible transitions is [26]:

$$P_{abcd}^1 = \frac{\delta_{ab}\delta_{cd}}{N^2 - 1}$$

$$P_{abcd}^{8ss} = \frac{N d_{abg} d_{cdg}}{N^2 - 4}$$

$$\begin{aligned}
 P_{abcd}^{8aa} &= \frac{f_{abg}f_{cdg}}{N} \\
 P_{abcd}^{10+\overline{10}} &= \frac{1}{2}(\delta_{ac}\delta_{bd} - \delta_{ad}\delta_{cb}) - \frac{f_{abg}f_{cdg}}{N} \\
 P_{abcd}^{27} &= \frac{Nd_{abg}d_{cdg}}{4(N+2)} + \frac{1}{2}f_{adg}f_{cbg} - \frac{1}{4}f_{abg}f_{cdg} + \frac{1}{4}\delta_{ad}\delta_{bc} + \frac{1}{4}\delta_{ac}\delta_{bd} + \frac{\delta_{ab}\delta_{cd}}{2(N+1)} \\
 P_{abcd}^0 &= -\frac{Nd_{abg}d_{cdg}}{4(N-2)} - \frac{1}{2}f_{adg}f_{cbg} + \frac{1}{4}f_{abg}f_{cdg} + \frac{1}{4}\delta_{ac}\delta_{bd} + \frac{1}{4}\delta_{ad}\delta_{cb} - \frac{\delta_{ab}\delta_{cd}}{2(N-1)} \\
 P_{abcd}^{8as} &= d_{cdg}if_{abg} + d_{abg}if_{cdg} \\
 P_{abcd}^{8sa} &= d_{cdg}if_{abg} - d_{abg}if_{cdg} \\
 P_{abcd}^{10-\overline{10}} &= \frac{1}{2}d_{acg}if_{bgd} - \frac{1}{2}d_{bgd}if_{acg}.
 \end{aligned} \tag{2.9}$$

Of these color tensors,  $P^1$ ,  $P^{8ss}$ ,  $P^{8aa}$ ,  $P^{10+\overline{10}}$ ,  $P^{27}$  and  $P^0$  are projectors, i.e. they satisfy  $P_{ABmn}^i P_{mnCD}^j = \delta_{ij} P_{ABCD}^i$  (no sum over  $i$ ). This also defines their normalizations. For three colors  $P^0$  annihilates anything. The last three tensors are not projectors, as can be seen from the fact that they have different symmetries w.r.t. interchanging the index pairs  $ab$  and  $cd$ . However  $P^{10-\overline{10}}$  still annihilates any non-decuplet or anti-decuplet state, and  $P^{8as}$  and  $P^{8sa}$  annihilate any non-octet state. The reason for using  $10 + \overline{10}$  and  $10 - \overline{10}$  is that  $10 + \overline{10}$  is symmetric w.r.t. interchanging quarks and anti-quarks in the root diagram.

The soft anomalous dimension matrix, in the basis eq. (2.9) is block diagonal. The color evolution matrix mixing the first 6 basis states (5 for  $N = 3$ ) can be stated

$$\mathbf{\Gamma}_{gg \rightarrow gg}^{6 \times 6} = \begin{pmatrix} -NT & 0 & N(U-V) & 0 & 0 & 0 \\ 0 & -\frac{1}{4}N(2T+U+V) & \frac{1}{4}N(U-V) & \frac{1}{2}N(U-V) & 0 & 0 \\ \frac{N(U-V)}{N^2-1} & \frac{N(U-V)}{4} & -\frac{N(2T+U+V)}{4} & 0 & \frac{N(N+3)(U-V)}{4(N+1)} & \frac{(N-3)N(U-V)}{4(N-1)} \\ 0 & \frac{N(U-V)}{N^2-4} & 0 & -\frac{N(U+V)}{2} & \frac{N(N+3)(U-V)}{4(N+2)} & \frac{(N-3)N(U-V)}{4(N-2)} \\ 0 & 0 & \frac{U-V}{N} & \frac{(N^2-N-2)(U-V)}{2N} & \frac{(2T-(N+1)(U+V))}{2} & 0 \\ 0 & 0 & \frac{U-V}{N} & \frac{(N^2+N-2)(U-V)}{2N} & 0 & \frac{(-2T-(N-1)(U+V))}{2} \end{pmatrix} \tag{2.10}$$

where again  $T = \Omega_{ab} + \Omega_{cd}$ ,  $U = \Omega_{ad} + \Omega_{bc}$  and  $V = \Omega_{ac} + \Omega_{bd}$ . The  $\Omega_{ij}$  factors are as in eq. (2.5), but with an extra minus sign for each outgoing gluon from the definition of the triple gluon vertex, which here, and throughout this paper, is taken to be

$$\begin{aligned}
 &f_{eig} \quad \text{with} \\
 &e = \text{the external (incoming or outgoing) eikonal gluon index} \\
 &i = \text{the internal (incoming or outgoing) eikonal gluon index} \\
 &g = \text{the soft exchange gluon index.}
 \end{aligned} \tag{2.11}$$

The advantage of using this convention is that the color algebra does not depend on how the diagram is drawn on a paper, and is also independent of whether a gluon is incoming or outgoing. The price to pay is an overall minus sign for each outgoing gluon compared to eq. (2.5). Tensors 7 to 9 form another decoupled diagonal block

$$\mathbf{\Gamma}_{gg \rightarrow gg}^{3 \times 3} = -\frac{N}{2} \times \text{Diagonal}[T+U, T+V, U+V]. \tag{2.12}$$

One way of understanding the decoupling of the  $6 \times 6$ -block is to note that the rules, in terms of  $f_{abc}$  and  $d_{abc}$ , that are used when calculating the effect of gluon exchange, have the property that they change the number of  $f$ s and  $d$ s only in units of two. The last three tensors in eq. (2.9), which have one  $d$  and one  $f$  will therefore decouple.

The above matrices would have been symmetric in an orthonormal basis. As the basis eq. (2.9) is not normalized, when calculating the cross section, the scalar product of the color vectors has to be used,  $\sigma = \mathbf{M}^\dagger \mathbf{S}_{gg \rightarrow gg} \mathbf{M}$  with

$$\mathbf{S}_{gg \rightarrow gg} = \text{Diagonal} \left[ 1, N^2 - 1, N^2 - 1, \frac{(N^2 - 1)(N^2 - 4)}{2}, \frac{N^2(N - 1)(N + 3)}{4}, \right. \\ \left. \frac{N^2(N + 1)(N - 3)}{4}, 2(N^2 - 4)(N^2 - 1), 2(N^2 - 4)(N^2 - 1), \frac{(N^2 - 4)(N^2 - 1)}{2} \right]. \quad (2.13)$$

### 3. $2 \rightarrow 3$ processes

For  $2 \rightarrow n$  scattering the method of mapping initial  $s$ -channel multiplets to final multiplets persists. However, the construction of the final multiplets is now more complicated, as more partons are to be combined. For the purpose of investigating the color structure of  $2 \rightarrow 3$  processes it is enough to consider

- a)  $q\bar{q} \rightarrow q\bar{q}g$
- b)  $gg \rightarrow q\bar{q}g$
- c)  $gg \rightarrow ggg$ .

The other cases can be obtained by changing incoming quarks to outgoing anti-quarks, vice versa, and by exchanging incoming gluons for outgoing gluons.

#### 3.1 Construction of complete orthogonal bases

Color conservation implies that if the incoming partons are in a certain color multiplet, then so must the outgoing partons be. As for the construction of possible multiplets for the incoming partons, this is not more complicated than in the  $2 \rightarrow 2$  case. An initial  $q\bar{q}$  pair could form a singlet or an octet, whereas 2 initial gluons may be in any of the states  $1, 8^a, 8^s, 10, \bar{10}, 27$ , or, for  $N \geq 4$ ,  $0$ .

The construction of final states is, however, more complicated as there are in general several ways of forming, for example an octet, out of the final partons. One way of systemizing the construction of the final state multiplets is to first consider the multiplet formed by two of the three outgoing partons. The multiplets  $M^1$  and  $M^2$  can together form a number of multiplets. Each of these, say  $M^{12}$ , can be combined with the remaining multiplet  $M^3$  to form an overall multiplet  $M^{\text{tot}}$ . Clearly the result  $M^{\text{tot}}$  has to match the multiplet of the incoming side. This general strategy has been applied to the above cases and will be detailed below. Note that this method of multiplet combination can be generalized to  $2 \rightarrow n$  processes by continuing the subgrouping of partons.



### 3.2 $q\bar{q} \rightarrow q\bar{q}g$

In the case of  $q_a\bar{q}_b \rightarrow q_c\bar{q}_d g_e$ , the initial  $q\bar{q}$  pair is either in a singlet or in an octet state. For an initial singlet, the final state must also be in a total singlet, meaning that the final  $q\bar{q}$  must form an octet to cancel the octet from the outgoing gluon, giving a color tensor proportional to  $\delta_{ab}t_{cd}^e$ .

If, on the other hand, the initial state is an octet, this can be matched by octets constructed in three different ways in the final state. The final  $q\bar{q}$  can form a singlet. In this case the initial octet is matched by the outgoing gluon,  $\delta_{cd}t_{ba}^e$ . If the final  $q\bar{q}$  is instead in an octet, this octet combined with the gluon octet, can form two different octets,  $8^a$  and  $8^s$ , which can match the incoming octet. For more than three colors there are no further states to be considered (clearly the gluon is then no longer an octet, but a multiplet of dimension  $N^2 - 1$ ). There are thus four color states in the basis. Guided by the multiplet decomposition, they may be written down as

$$\begin{aligned}
 T_{abcde}^1 &= \delta_{ab}t_{cd}^e \\
 T_{abcde}^2 &= \delta_{cd}t_{ba}^e \\
 T_{abcde}^3 &= t_{ba}^m t_{cd}^n f_{mne} \\
 T_{abcde}^4 &= t_{ba}^m t_{cd}^n d_{mne}.
 \end{aligned}
 \tag{3.1}$$

The normalization matrix can be found in appendix A.1, eq. (A.1).

### 3.3 $gg \rightarrow q\bar{q}g$

Similarly one can construct the basis for  $g_a g_b \rightarrow q_c \bar{q}_d g_e$ . In this case the initial two gluons can form any of the multiplets 1,  $8^a$ ,  $8^s$ , 10,  $\overline{10}$ , 27, 0 and this must be matched on the outgoing side. This can be done as:

$$\begin{aligned}
 &(ab)^1((cd)^8 e)^1 \\
 &(ab)^{8^a}((cd)^1 e)^8 \\
 &(ab)^{8^s}((cd)^1 e)^8 \\
 &(ab)^{8^a}((cd)^8 e)^{8^a} \\
 &(ab)^{8^s}((cd)^8 e)^{8^a} \\
 &(ab)^{8^a}((cd)^8 e)^{8^s} \\
 &(ab)^{8^s}((cd)^8 e)^{8^s} \\
 &(ab)^{10}((cd)^8 e)^{10} \\
 &(ab)^{\overline{10}}((cd)^8 e)^{\overline{10}} \\
 &(ab)^{27}((cd)^8 e)^{27} \\
 &(ab)^0((cd)^8 e)^0.
 \end{aligned}
 \tag{3.2}$$

From this it may be concluded that there are 11 independent color tensors. If the outgoing  $q\bar{q}$  pair would be in an octet, there would be 9 color tensors as for  $gg \rightarrow gg$ , but since the  $q\bar{q}$  pair can also form a singlet there are two extra states, corresponding to the second

and third case above. The explicit tensors are, together with the normalizations, given in appendix A.2, eq. (A.2) and eq. (A.3).

### 3.4 $gg \rightarrow ggg$

By the same principle, the basis for  $gg \rightarrow ggg$  can be constructed. This is a lengthy exercise as there turns out to be 44 independent color tensors. Fortunately, due to the reasons explained in section 2.4, the anomalous dimension matrix factorizes into a 22-block, containing an odd number of  $f$ s (and an even number of  $d$ s), and another 22-block, containing an even number of  $f$ s (and an odd number of  $d$ s).

While combining the multiplet of  $cd$  with the octet of  $e$ , the decompositions of  $10 \otimes 8$ ,  $\overline{10} \otimes 8$ ,  $27 \otimes 8$  and  $0 \otimes 8$  are needed. These may be obtained using Young tableaux. A complication occurs for the “decuplets” when  $N \geq 4$ . Apart from the extra 0-plets which can be combined to form tensors, there is also an extra “decuplet” in  $8 \otimes 10$ , and an extra “anti-decuplet” in  $8 \otimes \overline{10}$ . Like for  $gg \rightarrow gg$  the non-symmetric decuplets 10 and  $\overline{10}$  are combined as  $P^{10+\overline{10}}$  and  $P^{10-\overline{10}}$ .

To explicitly see that the color tensors, written down in eq. (A.4), are orthogonal, consider for example  $T_{abcde}^{17}$ . This tensor is orthogonal to the tensors  $T^1 - T^{16}$  and  $T^{20} - T^{22}$  since in  $T^{17}$  the gluons  $ab$  are projected onto the 27-plet whereas in  $T^1 - T^{16}$  and  $T^{20} - T^{22}$   $ab$  are projected into a different multiplet.  $T^{17}$  is also orthogonal to  $T^{18}$  and  $T^{19}$  due to the different  $cd$  projectors. Similarly one can argue about the orthogonality of all of the tensors  $T^1$  and  $T^{17} - T^{22}$ . To see that all the octets ( $T^2 - T^{10}$ ) are mutually orthogonal one may use the relation  $f_{ABm}d_{ABn} = 0$ , following from the different symmetries w.r.t.  $ab$ . For the decuplet tensors  $T^{11} - T^{16}$ , the orthogonality of the first two and the last four follows from the projector argument, whereas tensor 14 is manually constructed to be orthogonal to tensor 13, which is the only decuplet for  $N = 3$ . Similarly the bases eq. (3.1) and eq. (A.2) can be seen to be orthogonal.

## 4. Results

The soft anomalous dimension matrices have been calculated for the cases  $q\bar{q} \rightarrow q\bar{q}g$ ,  $q\bar{q} \rightarrow ggg$  and  $gg \rightarrow ggg$  using the aforementioned  $s$ -channel bases. However, the results are applicable to any colored five parton process by a simple exchange of incoming and outgoing particles.

The sign convention for the triple gluon vertex  $f_{abc}$  is calculated using the index order from eq. (2.12) everywhere, implying an extra overall minus sign for each outgoing gluon, compared to eq. (2.5). (For a topology in which radiation is forbidden within a central rapidity region  $Y$  and all outgoing partons have rapidity  $> Y$ , the momentum integrals of [8] can be used provided care is taken to compensate for different sign conventions.)

The results presented here are calculated using a Mathematica document which is electronically attached. It has been checked that the resulting matrices are symmetric in normalized bases [27, 28]. It has also been explicitly checked that the bases used are complete by projecting the color tensor arising after each possible exchange into the basis and checking the the norm of the projected tensor is the same as the norm of the original

tensor. Furthermore the same piece of code has been checked to, modulo sign conventions, reproduce the results for several of the  $2 \rightarrow 2$  cases and for  $qq \rightarrow q\bar{q}g$  in [8].

The results are explicitly stated in appendix B, but a few general remarks are in place. First of all, note that any gluon exchange between the partons  $ab$  must result in the same multiplet for  $ab$ , and also does not affect the  $cde$  indices. Thus, an exchange between  $ab$  will result in a diagonal matrix in all bases under consideration here. Similarly a gluon exchange between  $bc$  does not change the tensor. These gluon exchanges therefore represent diagonal contributions to the soft anomalous dimension matrices.

An exchange between the partons  $ce$  or  $de$ , does not change the overall multiplet (as the initial state  $ab$  is unaffected), but it may, for example, change a final state octet to a different final state octet. These contributions to  $\mathbf{\Gamma}$  are thus block diagonal. There is, in principle, a similar symmetry w.r.t. gluon exchange between  $ae$  or  $be$  as this does not change the multiplet of  $cd$ . However, due to the ordering of basis vectors, this does not manifest itself in a block diagonal structure. That the above simplifications are present in the computer algebra results offers yet another consistency check.

## 5. Conclusions

The color structure needed for resummation of all colored five parton processes has been calculated. The result is a  $4 \times 4$ -matrix for processes involving four external quarks or anti-quarks and one gluon, an  $11 \times 11$ -matrix for processes involving two external quarks or anti-quarks and three gluons, and a  $22 \times 22$ -matrix for five gluon processes.

The method presented here can, by using higher projection operators, easily be extended to processes with even more final state colored particles. However, the calculations, although performed by a computer, are quite demanding. To accomplish significant further progress additional theoretical insight is probably needed.

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## A. Basis tensors and normalizations

### A.1 $q\bar{q} \rightarrow q\bar{q}g$

The basis tensors for  $q_a\bar{q}_b \rightarrow q_c\bar{q}_d g_e$  are stated in eq. (3.1). The normalizations are

$$\mathbf{S}_{q\bar{q} \rightarrow q\bar{q}g} = \text{Diagonal} \left[ \frac{1}{2}N(N^2 - 1), \frac{1}{2}N(N^2 - 1), \frac{1}{4}N(N^2 - 1), \frac{(N^2 - 4)(N^2 - 1)}{4N} \right]. \tag{A.1}$$

### A.2 $gg \rightarrow q\bar{q}g$

To explicitly write down the tensors in eq. (3.2), the projectors in eq. (2.9) can be used together with a generator component  $t_{cd}^g$  to denote the  $q\bar{q}$  pair in an octet. The basis tensors for  $g_a g_b \rightarrow q_c \bar{q}_d g_e$  may thus be chosen as:

$$\begin{aligned}
 T_{abcde}^1 &= t_{cd}^e \delta_{ab} \\
 T_{abcde}^2 &= i f_{abe} \delta_{cd} \\
 T_{abcde}^3 &= d_{abe} \delta_{cd} \\
 T_{abcde}^4 &= i f_{abn} i f_{men} t_{cd}^m \\
 T_{abcde}^5 &= d_{abn} i f_{men} t_{cd}^m \\
 T_{abcde}^6 &= i f_{abn} d_{men} t_{cd}^m \\
 T_{abcde}^7 &= d_{abn} d_{men} t_{cd}^m \\
 T_{abcde}^8 &= P_{abme}^{10+\bar{10}} t_{cd}^m \\
 T_{abcde}^9 &= P_{abme}^{10-\bar{10}} t_{cd}^m \\
 T_{abcde}^{10} &= -P_{abme}^{27} t_{cd}^m \\
 T_{abcde}^{11} &= P_{abme}^0 t_{cd}^m,
 \end{aligned} \tag{A.2}$$

where the symmetrized and anti-symmetrized decuplet projectors  $P^{10+\bar{10}}$  and  $P^{10-\bar{10}}$  are used instead of  $P^{10}$  and  $P^{\bar{10}}$ . The scalar products of these basis tensors are given by:

$$\mathbf{S}_{gg \rightarrow q\bar{q}g} = \text{Diagonal} \left[ \begin{aligned} & \frac{(N^2-1)^2}{2}, N^2(N^2-1), (N^2-1)(N^2-4), \frac{N^2(N^2-1)}{2}, \frac{(N^2-4)(N^2-1)}{2}, \frac{(N^2-4)(N^2-1)}{2}, \\ & \frac{(N^2-4)^2(N^2-1)}{2N^2}, \frac{(N^2-4)(N^2-1)}{4}, \frac{(N^2-4)(N^2-1)}{4}, \frac{N^2(N-1)(N+3)}{8}, \frac{N^2(N+1)(N-3)}{8} \end{aligned} \right]. \tag{A.3}$$

### A.3 $gg \rightarrow ggg$

For the  $g_a g_b \rightarrow g_c g_d g_e$  case the physically relevant tensors are chosen as:

$$\begin{aligned}
 T_{abcde}^1 &= i f_{cde} \delta_{ab} \\
 T_{abcde}^2 &= i f_{abe} \delta_{cd} \\
 T_{abcde}^3 &= -i f_{abl} f_{cdm} f_{elm} \\
 T_{abcde}^4 &= i d_{cdm} d_{elm} f_{abl} \\
 T_{abcde}^5 &= i d_{abl} d_{elm} f_{cdm} \\
 T_{abcde}^6 &= i d_{abl} d_{cdm} f_{elm} \\
 T_{abcde}^7 &= i f_{abn} P_{cden}^{10+\bar{10}} \\
 T_{abcde}^8 &= d_{abn} P_{cden}^{10-\bar{10}}
 \end{aligned}$$

$$\begin{aligned}
 T_{abcde}^9 &= if_{anb}P^{27}_{cde} \\
 T_{abcde}^{10} &= if_{abn}P^0_{cde} \\
 T_{abcde}^{11} &= if_{cdn}P^{10+\overline{10}}_{neab} \\
 T_{abcde}^{12} &= d_{cdn}P^{10-\overline{10}}_{neab} \\
 T_{abcde}^{13} &= if_{len}P^{10+\overline{10}}_{abmn}P^{10+\overline{10}}_{cdlm} \\
 T_{abcde}^{14} &= d_{len}P^{10-\overline{10}}_{abmn}P^{10+\overline{10}}_{cdlm} + \frac{1}{N}if_{len}P^{10+\overline{10}}_{abmn}P^{10+\overline{10}}_{cdlm} \\
 T_{abcde}^{15} &= if_{enl}P^{27}_{cdlm}P^{10+\overline{10}}_{abmn} \\
 T_{abcde}^{16} &= if_{eln}P^{10+\overline{10}}_{abmn}P^0_{cdlm} \\
 T_{abcde}^{17} &= if_{cnd}P^{27}_{neab} \\
 T_{abcde}^{18} &= if_{enl}P^{27}_{abmn}P^{10+\overline{10}}_{cdlm} \\
 T_{abcde}^{19} &= if_{eln}P^{27}_{abmn}P^{27}_{cdlm} \\
 T_{abcde}^{20} &= if_{cdn}P^0_{abne} \\
 T_{abcde}^{21} &= if_{eln}P^{10+\overline{10}}_{cdlm}P^0_{abmn} \\
 T_{abcde}^{22} &= if_{eln}P^0_{abmn}P^0_{cdlm}
 \end{aligned} \tag{A.4}$$

There is also a decoupled block with equally many basis tensors. The tensors in this block can be obtained by  $if \leftrightarrow d$  in all tensors above except  $T^{14}$  in which the coefficient  $1/N$  in front of the second term also has to be replaced by  $N/(N^2 - 8)$ .

The normalization matrix is:

$\mathbf{S}_{gg \rightarrow ggg} = \text{Diagonal}$

$$\begin{aligned}
 &[ N(N^2 - 1)^2, N(N^2 - 1)^2, N^3(N^2 - 1), \frac{(N^2 - 4)^2(N^2 - 1)}{N}, \frac{(N^2 - 4)^2(N^2 - 1)}{N}, \\
 &\frac{(N^2 - 4)^2(N^2 - 1)}{N}, \frac{1}{2}N(N^2 - 1)(N^2 - 4), \frac{(N^2 - 4)^2(N^2 - 1)}{2N}, \frac{1}{4}N^3(N - 1)(N + 3), \\
 &\frac{1}{4}N^3(N + 1)(N - 3), \frac{1}{2}N(N^2 - 1)(N^2 - 4), \frac{(N^2 - 4)^2(N^2 - 1)}{2N}, \frac{1}{4}N(N^2 - 1)(N^2 - 4), \\
 &\frac{(N^2 - 9)(N^2 - 4)(N^2 - 1)}{4N}, \frac{1}{8}N(N^2 - 1)(N + 3)(N - 2), \frac{1}{8}N(N^2 - 1)(N + 2)(N - 3), \\
 &\frac{1}{4}N^3(N + 3)(N - 1), \frac{1}{8}N(N^2 - 1)(N + 3)(N - 2), \frac{1}{8}N^2(N^2 - 1)(N + 3), \\
 &\frac{1}{4}N^3(N + 1)(N - 3), \frac{1}{8}N(N^2 - 1)(N + 2)(N - 3), \frac{1}{8}N^2(N^2 - 1)(N - 3) ].
 \end{aligned} \tag{A.5}$$

## B. Algebraic results

Below are the algebraic results for the color part of the anomalous dimension matrix in the notation  $p_1 p_2 \rightarrow p_3 p_4 p_5$  for the processes  $q\bar{q} \rightarrow q\bar{q}g$ ,  $gg \rightarrow q\bar{q}g$  and  $gg \rightarrow ggg$ .

### B.1 Results for $q\bar{q} \rightarrow q\bar{q}g$

The anomalous dimension matrix, using the basis eq. (3.1) is calculated to be:

$$\begin{aligned}
 \Gamma_{q\bar{q} \rightarrow q\bar{q}g} = & \begin{pmatrix} \frac{N^2-1}{2N} & 0 & 0 & 0 \\ 0 & -\frac{1}{2N} & 0 & 0 \\ 0 & 0 & -\frac{1}{2N} & 0 \\ 0 & 0 & 0 & -\frac{1}{2N} \end{pmatrix} \Omega_{12} + \begin{pmatrix} -\frac{1}{2N} & 0 & 0 & 0 \\ 0 & \frac{N^2-1}{2N} & 0 & 0 \\ 0 & 0 & -\frac{1}{2N} & 0 \\ 0 & 0 & 0 & -\frac{1}{2N} \end{pmatrix} \Omega_{34} \\
 & + \begin{pmatrix} 0 & \frac{1}{2N} & -\frac{1}{4} & \frac{1}{4} - \frac{1}{N^2} \\ \frac{1}{2N} & 0 & -\frac{1}{4} & \frac{1}{4} - \frac{1}{N^2} \\ -\frac{1}{2} & -\frac{1}{2} & \frac{N^2-2}{4N} & \frac{1}{N} - \frac{N}{4} \\ \frac{1}{2} & \frac{1}{2} & -\frac{N}{4} & \frac{N^2-6}{4N} \end{pmatrix} \Omega_{13} + \begin{pmatrix} 0 & \frac{1}{2N} & \frac{1}{4} & \frac{1}{4} - \frac{1}{N^2} \\ \frac{1}{2N} & 0 & -\frac{1}{4} & \frac{1}{4} - \frac{1}{N^2} \\ \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2N} & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & -\frac{3}{2N} \end{pmatrix} \Omega_{14} \\
 & + \begin{pmatrix} 0 & \frac{1}{2N} & -\frac{1}{4} & \frac{1}{4} - \frac{1}{N^2} \\ \frac{1}{2N} & 0 & \frac{1}{4} & \frac{1}{4} - \frac{1}{N^2} \\ -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2N} & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & -\frac{3}{2N} \end{pmatrix} \Omega_{23} + \begin{pmatrix} 0 & \frac{1}{2N} & \frac{1}{4} & \frac{1}{4} - \frac{1}{N^2} \\ \frac{1}{2N} & 0 & \frac{1}{4} & \frac{1}{4} - \frac{1}{N^2} \\ \frac{1}{2} & \frac{1}{2} & \frac{N^2-2}{4N} & \frac{N}{4} - \frac{1}{N} \\ \frac{1}{2} & \frac{1}{2} & \frac{N}{4} & \frac{N^2-6}{4N} \end{pmatrix} \Omega_{24} \\
 & + \begin{pmatrix} 0 & 0 & -\frac{1}{2} & 0 \\ 0 & -\frac{N}{2} & 0 & 0 \\ -1 & 0 & -\frac{N}{4} & \frac{1}{N} - \frac{N}{4} \\ 0 & 0 & -\frac{N}{4} & -\frac{N}{4} \end{pmatrix} \Omega_{15} + \begin{pmatrix} 0 & 0 & -\frac{1}{2} & 0 \\ 0 & \frac{N}{2} & 0 & 0 \\ -1 & 0 & \frac{N}{4} & \frac{1}{N} - \frac{N}{4} \\ 0 & 0 & -\frac{N}{4} & \frac{N}{4} \end{pmatrix} \Omega_{25} \\
 & + \begin{pmatrix} \frac{N}{2} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 1 & \frac{N}{4} & \frac{N}{4} - \frac{1}{N} \\ 0 & 0 & \frac{N}{4} & \frac{N}{4} \end{pmatrix} \Omega_{35} + \begin{pmatrix} -\frac{N}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 1 & -\frac{N}{4} & \frac{N}{4} - \frac{1}{N} \\ 0 & 0 & \frac{N}{4} & -\frac{N}{4} \end{pmatrix} \Omega_{45} \tag{B.1}
 \end{aligned}$$

The momentum integrals  $\Omega_{ij}$  are as in eq. (2.5), including the overall minus sign for exchanges involving the anti-quark or the final state gluon.

Note the diagonal form of the exchange between parton pairs in definite multiplets, that is the 12 and 34 components of  $\Gamma_{q\bar{q} \rightarrow q\bar{q}g}$ . Also, note the block diagonal form of the exchange between 35 and 45.

### B.2 Result for $gg \rightarrow q\bar{q}g$

Below the results for the soft anomalous dimension matrix

$$\mathbf{\Gamma}_{gg \rightarrow q\bar{q}g} = \sum_{i < j} \Omega_{ij} C_{gg \rightarrow q\bar{q}g}^{ij} \tag{B.2}$$

are stated in terms of the basis eq. (A.2). The  $\Omega_{ij}$  factors are given by eq. (2.5), including the overall minus sign for each involved anti-quark, and the overall minus sign for the outgoing gluon (arising due to the convention for the triple gluon vertex, eq. (2.12), used in  $C_{gg \rightarrow q\bar{q}g}^{ij}$ ).

$$C_{gg \rightarrow q\bar{q}g}^{12} = \text{Diagonal} \left[ -N, -\frac{N}{2}, -\frac{N}{2}, -\frac{N}{2}, -\frac{N}{2}, -\frac{N}{2}, -\frac{N}{2}, 0, 0, 1, -1 \right] \tag{B.3}$$

$$C_{gg \rightarrow q\bar{q}g}^{34} = \text{Diagonal} \left[ -\frac{1}{2N}, \frac{N^2-1}{2N}, \frac{N^2-1}{2N}, -\frac{1}{2N}, -\frac{1}{2N}, -\frac{1}{2N}, -\frac{1}{2N}, -\frac{1}{2N}, -\frac{1}{2N}, -\frac{1}{2N}, -\frac{1}{2N} \right] \quad (\text{B.4})$$

$$C_{gg \rightarrow q\bar{q}g}^{35} = \begin{pmatrix} \frac{N}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & \frac{N}{4} & 0 & \frac{N}{4} - \frac{1}{N} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & \frac{N}{4} & 0 & \frac{N}{4} - \frac{1}{N} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{N}{4} & 0 & \frac{N}{4} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{N}{4} & 0 & \frac{N}{4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix} \quad (\text{B.5})$$

$$C_{gg \rightarrow q\bar{q}g}^{45} = \begin{pmatrix} -\frac{N}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -\frac{N}{4} & 0 & \frac{N}{4} - \frac{1}{N} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -\frac{N}{4} & 0 & \frac{N}{4} - \frac{1}{N} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{N}{4} & 0 & -\frac{N}{4} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{N}{4} & 0 & -\frac{N}{4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} \end{pmatrix} \quad (\text{B.6})$$

$$C_{gg \rightarrow q\bar{q}g}^{15} = \begin{pmatrix} 0 & 0 & 0 & -\frac{N^2}{N^2-1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{N}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{N}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & -\frac{N}{4} & 0 & 0 & \frac{1}{N} - \frac{N}{4} & 0 & 0 & \frac{N+3}{4N+4} & -\frac{N-3}{4(N-1)} \\ 0 & 0 & 0 & 0 & -\frac{N}{4} & -\frac{N}{4} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{N}{4} & -\frac{N}{4} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{N^3}{16-4N^2} & 0 & 0 & -\frac{N}{4} & \frac{N^2}{2(N^2-4)} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -\frac{N}{2} & 0 & -\frac{N(N+3)}{4(N+2)} & \frac{(N-3)N}{4(N-2)} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{N}{2} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -\frac{N}{2} + \frac{1}{2} + \frac{1}{N} & 0 & \frac{1}{2}(-N-1) & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & \frac{N^2+N-2}{2N} & 0 & 0 & \frac{1-N}{2} \end{pmatrix} \quad (\text{B.7})$$

$$C_{gg \rightarrow q\bar{q}g}^{25} = \begin{pmatrix} 0 & 0 & 0 & \frac{N^2}{N^2-1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{N}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{N}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -\frac{N}{4} & 0 & 0 & \frac{N}{4} - \frac{1}{N} & 0 & 0 & -\frac{N+3}{4N+4} & \frac{N-3}{4(N-1)} \\ 0 & 0 & 0 & 0 & -\frac{N}{4} & \frac{N}{4} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{N}{4} & -\frac{N}{4} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{N^3}{4(N^2-4)} & 0 & 0 & -\frac{N}{4} & \frac{N^2}{8-2N^2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & -\frac{N}{2} & 0 & \frac{N(N+3)}{4(N+2)} & -\frac{(N-3)N}{4(N-2)} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{N}{2} & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & \frac{1}{2}(N-1-\frac{2}{N}) & 0 & \frac{1}{2}(-N-1) & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -\frac{N}{2} - \frac{1}{2} + \frac{1}{N} & 0 & 0 & \frac{1-N}{2} \end{pmatrix} \quad (\text{B.8})$$

$$C_{gg \rightarrow q\bar{q}g}^{13} = \begin{pmatrix} 0 & \frac{N}{N^2-1} & 0 & \frac{N^2}{2-2N^2} & 0 & \frac{N^2-4}{2(N^2-1)} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2N} & 0 & 0 & -\frac{1}{4} & 0 & 0 & \frac{1}{4} - \frac{1}{N^2} & 0 & 0 & \frac{N+3}{8N+8} & \frac{N-3}{8(N-1)} \\ 0 & 0 & 0 & 0 & -\frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 \\ -\frac{1}{2} & -\frac{1}{2} & 0 & \frac{N}{8} & \frac{N^2-4}{8N} & -\frac{N^2-4}{8N} & -\frac{N^2-4}{8N} & 0 & 0 & \frac{N+3}{8N+8} & -\frac{N-3}{8(N-1)} \\ 0 & 0 & -\frac{1}{2} & \frac{N}{8} & \frac{N}{8} & -\frac{N}{8} & -\frac{N^2-4}{8N} & \frac{1}{4} & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & -\frac{N}{8} & -\frac{N}{8} & \frac{N}{8} & \frac{N^2-12}{8N} & 0 & 0 & \frac{N(N+3)}{8(N^2+3N+2)} & -\frac{(N-3)N}{8(N^2-3N+2)} \\ 0 & \frac{N^2}{2(N^2-4)} & 0 & \frac{N^3}{32-8N^2} & -\frac{N}{8} & \frac{N(N^2-12)}{8(N^2-4)} & \frac{N}{8} & \frac{N^2}{4(N^2-4)} & \frac{N}{8-2N^2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & \frac{N}{4} & -\frac{1}{4} & -\frac{N(N+3)}{8(N+2)} & \frac{(N-3)N}{8(N-2)} \\ 0 & 0 & 1 & 0 & 0 & 0 & -\frac{1}{N} & -\frac{1}{4} & \frac{N}{4} & -\frac{N(N+3)}{8(N+2)} & -\frac{(N-3)N}{8(N-2)} \\ 0 & 1 & 0 & \frac{1}{2} & 0 & \frac{1}{2} - \frac{1}{N} & 0 & \frac{-N^2+N+2}{4N} & \frac{-N^2+N+2}{4N} & \frac{N+1}{4} & 0 \\ 0 & 1 & 0 & -\frac{1}{2} & 0 & -\frac{N+2}{2N} & 0 & \frac{N^2+N-2}{4N} & \frac{1}{4}(-N-1+\frac{2}{N}) & 0 & \frac{N-1}{4} \end{pmatrix} \quad (\text{B.9})$$



$$C_{gg \rightarrow q\bar{q}g}^{23} = \begin{pmatrix} 0 & -\frac{N}{N^2-1} & 0 & \frac{N^2}{2(N^2-1)} & 0 & -\frac{N^2-4}{2(N^2-1)} & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{2N} & 0 & 0 & -\frac{1}{4} & 0 & 0 & \frac{1}{N^2} - \frac{1}{4} & 0 & 0 & -\frac{N+3}{8N+8} & -\frac{N-3}{8(N-1)} \\ 0 & 0 & 0 & 0 & -\frac{1}{4} & -\frac{1}{4} & 0 & 0 & -\frac{1}{4} & 0 & 0 \\ \frac{1}{2} & -\frac{1}{2} & 0 & \frac{N}{8} & -\frac{N^2-4}{8N} & -\frac{N^2-4}{8N} & \frac{N^2-4}{8N} & 0 & 0 & -\frac{N+3}{8N+8} & \frac{N-3}{8(N-1)} \\ 0 & 0 & -\frac{1}{2} & -\frac{N}{8} & \frac{N}{8} & \frac{N}{8} & -\frac{N^2-4}{8N} & -\frac{1}{4} & 0 & 0 & 0 \\ -\frac{1}{2} & 0 & -\frac{1}{2} & -\frac{N}{8} & \frac{N}{8} & \frac{N}{8} & \frac{3}{2N} - \frac{N}{8} & 0 & 0 & -\frac{N(N+3)}{8(N^2+3N+2)} & \frac{(N-3)N}{8(N^2-3N+2)} \\ 0 & \frac{N^2}{8-2N^2} & 0 & \frac{N^3}{8(N^2-4)} & -\frac{N}{8} & -\frac{N(N^2-12)}{8(N^2-4)} & \frac{N}{8} & \frac{N^2}{16-4N^2} & \frac{N}{2(N^2-4)} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & -\frac{1}{2} & \frac{N}{4} & -\frac{1}{4} & \frac{N(N+3)}{8(N+2)} & -\frac{(N-3)N}{8(N-2)} \\ 0 & 0 & -1 & 0 & 0 & 0 & \frac{1}{N} & -\frac{1}{4} & \frac{N}{4} & \frac{N(N+3)}{8(N+2)} & \frac{(N-3)N}{8(N-2)} \\ 0 & -1 & 0 & -\frac{1}{2} & 0 & \frac{1}{N} - \frac{1}{2} & 0 & -\frac{-N^2+N+2}{4N} & -\frac{-N^2+N+2}{4N} & \frac{N+1}{4} & 0 \\ 0 & -1 & 0 & \frac{1}{2} & 0 & \frac{1}{2} + \frac{1}{N} & 0 & \frac{1}{4}(-N-1 + \frac{2}{N}) & \frac{N^2+N-2}{4N} & 0 & \frac{N-1}{4} \end{pmatrix} \quad (\text{B.10})$$

$$C_{gg \rightarrow q\bar{q}g}^{14} = \begin{pmatrix} 0 & \frac{N}{N^2-1} & 0 & \frac{N^2}{2(N^2-1)} & 0 & \frac{N^2-4}{2(N^2-1)} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2N} & 0 & 0 & -\frac{1}{4} & 0 & 0 & \frac{1}{4} - \frac{1}{N^2} & 0 & 0 & \frac{N+3}{8N+8} & \frac{N-3}{8(N-1)} \\ 0 & 0 & 0 & 0 & -\frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 \\ \frac{1}{2} & -\frac{1}{2} & 0 & -\frac{N}{8} & \frac{N^2-4}{8N} & -\frac{N^2-4}{8N} & \frac{N^2-4}{8N} & 0 & 0 & -\frac{N+3}{8N+8} & \frac{N-3}{8(N-1)} \\ 0 & 0 & -\frac{1}{2} & \frac{N}{8} & -\frac{N}{8} & \frac{N}{8} & -\frac{N^2-4}{8N} & \frac{1}{4} & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & -\frac{N}{8} & \frac{N}{8} & -\frac{N}{8} & \frac{N^2-12}{8N} & 0 & 0 & \frac{N(N+3)}{8(N^2+3N+2)} & -\frac{(N-3)N}{8(N^2-3N+2)} \\ 0 & \frac{N^2}{2(N^2-4)} & 0 & \frac{N^3}{8(N^2-4)} & -\frac{N}{8} & \frac{N(N^2-12)}{8(N^2-4)} & -\frac{N}{8} & \frac{N^2}{16-4N^2} & \frac{N}{8-2N^2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & -\frac{1}{2} & -\frac{N}{4} & -\frac{1}{4} & \frac{N(N+3)}{8(N+2)} & -\frac{(N-3)N}{8(N-2)} \\ 0 & 0 & 1 & 0 & 0 & 0 & -\frac{1}{N} & -\frac{1}{4} & -\frac{N}{4} & -\frac{N(N+3)}{8(N+2)} & -\frac{(N-3)N}{8(N-2)} \\ 0 & 1 & 0 & -\frac{1}{2} & 0 & \frac{1}{2} - \frac{1}{N} & 0 & -\frac{-N^2+N+2}{4N} & \frac{-N^2+N+2}{4N} & \frac{1}{4}(-N-1) & 0 \\ 0 & 1 & 0 & \frac{1}{2} & 0 & -\frac{N+2}{2N} & 0 & \frac{1}{4}(-N-1 + \frac{2}{N}) & \frac{1}{4}(-N-1 + \frac{2}{N}) & 0 & \frac{1-N}{4} \end{pmatrix} \quad (\text{B.11})$$

$$C_{gg \rightarrow q\bar{q}g}^{24} = \begin{pmatrix} 0 & -\frac{N}{N^2-1} & 0 & \frac{N^2}{2-2N^2} & 0 & -\frac{N^2-4}{2(N^2-1)} & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{2N} & 0 & 0 & -\frac{1}{4} & 0 & 0 & \frac{1}{N^2} - \frac{1}{4} & 0 & 0 & -\frac{N+3}{8N+8} & -\frac{N-3}{8(N-1)} \\ 0 & 0 & 0 & 0 & -\frac{1}{4} & -\frac{1}{4} & 0 & 0 & -\frac{1}{4} & 0 & 0 \\ -\frac{1}{2} & -\frac{1}{2} & 0 & -\frac{N}{8} & -\frac{N^2-4}{8N} & -\frac{N^2-4}{8N} & -\frac{N^2-4}{8N} & 0 & 0 & \frac{N+3}{8N+8} & -\frac{N-3}{8(N-1)} \\ 0 & 0 & -\frac{1}{2} & -\frac{N}{8} & -\frac{N}{8} & -\frac{N}{8} & -\frac{N^2-4}{8N} & -\frac{1}{4} & 0 & 0 & 0 \\ -\frac{1}{2} & 0 & -\frac{1}{2} & -\frac{N}{8} & -\frac{N}{8} & -\frac{N}{8} & \frac{3}{2N} - \frac{N}{8} & 0 & 0 & -\frac{N(N+3)}{8(N^2+3N+2)} & \frac{(N-3)N}{8(N^2-3N+2)} \\ 0 & \frac{N^2}{8-2N^2} & 0 & \frac{N^3}{32-8N^2} & -\frac{N}{8} & -\frac{N(N^2-12)}{8(N^2-4)} & -\frac{N}{8} & \frac{N^2}{4(N^2-4)} & \frac{N}{2(N^2-4)} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & \frac{1}{2} & -\frac{N}{4} & -\frac{1}{4} & -\frac{N(N+3)}{8(N+2)} & \frac{(N-3)N}{8(N-2)} \\ 0 & 0 & -1 & 0 & 0 & 0 & \frac{1}{N} & -\frac{1}{4} & -\frac{N}{4} & \frac{N(N+3)}{8(N+2)} & \frac{(N-3)N}{8(N-2)} \\ 0 & -1 & 0 & \frac{1}{2} & 0 & \frac{1}{N} - \frac{1}{2} & 0 & \frac{-N^2+N+2}{4N} & \frac{-N^2+N+2}{4N} & \frac{1}{4}(-N-1) & 0 \\ 0 & -1 & 0 & -\frac{1}{2} & 0 & \frac{1}{2} + \frac{1}{N} & 0 & \frac{N^2+N-2}{4N} & \frac{N^2+N-2}{4N} & 0 & \frac{1-N}{4} \end{pmatrix} \quad (\text{B.12})$$



$$C_{gg \rightarrow ggg}^{25} = \quad (B.17)$$

0	0	$\frac{N^2}{-1-N^2}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	$-\frac{N}{2}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	$-\frac{N}{4}$	0	$\frac{-4+N^2}{4N}$	0	0	0	0	0	0	0	$\frac{-3-N}{4(1-N)}$	0	0	$\frac{-3-N}{4(1-N)}$	0	0	0
0	0	0	$-\frac{N}{4}$	0	$\frac{N}{4}$	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	$\frac{N^2}{4(-4+N^2)}$	0	$-\frac{N}{4}$	0	0	0	$-\frac{N^2}{2(-4+N^2)}$	0	0	0	0	0	0	0	0	0	0
0	0	0	$\frac{N}{4}$	0	$-\frac{N}{4}$	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	$-\frac{1}{2}$	0	0	0	0	0	0	$\frac{-3-N}{4(2+N)}$	0	$\frac{3-N}{4(-2+N)}$	0	0
0	0	0	0	0	0	$-\frac{N^2}{2(-4+N^2)}$	0	0	0	0	$\frac{N^2}{2(-4+N^2)}$	0	0	0	0	0	0	0
0	0	0	0	0	0	0	$\frac{1}{2}$	0	0	0	0	0	0	0	$\frac{-1-N}{2N}$	0	0	0
0	0	0	0	0	0	0	$-\frac{1}{2}$	0	0	0	0	0	0	0	0	0	$\frac{1-N}{2N}$	0
0	0	0	0	-1	0	0	0	$-\frac{N}{2}$	0	0	0	0	0	$\frac{N(3+N)}{4(2+N)}$	0	$-\frac{(-3+N)N}{4(-2+N)}$	0	0
0	0	0	0	0	0	0	0	0	0	$-\frac{N}{2}$	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	1	0	0	0	$-\frac{N}{4}$	0	0	$-\frac{(-2+N)(3+N)}{8(2+N)}$	0	$\frac{(-3+N)(2+N)}{8(-2+N)}$	0	
0	0	0	0	0	0	0	0	0	0	$-\frac{N}{4}$	0	0	0	$-\frac{N}{8}$	0	$-\frac{N}{8}$	0	0
0	0	0	0	0	0	0	0	0	0	0	0	$\frac{2+N}{4}$	0	0	$\frac{2+N}{4}$	0	0	0
0	0	0	0	0	0	0	0	0	0	0	$\frac{1}{4}(-2-N)$	0	0	0	0	0	$\frac{2-N}{4}$	0
0	0	-1	0	0	0	0	0	$\frac{-2-N+N^2}{2N}$	0	0	0	0	$\frac{1}{2}(-1-N)$	0	0	0	0	0
0	0	0	0	0	0	-1	0	0	0	$\frac{2-N}{4}$	$-\frac{(-3+N)(2+N)}{4N}$	0	0	$\frac{1}{4}(-2-N)$	0	0	0	0
0	0	0	0	0	0	0	-1	0	0	0	$\frac{-4+N^2}{4N}$	0	0	0	$-\frac{N}{4}$	0	0	0
0	0	1	0	0	0	0	0	$\frac{2-N+N^2}{2N}$	0	0	0	0	0	0	$\frac{1-N}{2}$	0	0	0
0	0	0	0	0	0	-1	0	0	0	$\frac{2+N}{4}$	$-\frac{(-2+N)(3+N)}{4N}$	0	0	0	0	0	$\frac{2-N}{4}$	0
0	0	0	0	0	0	0	0	-1	0	0	0	$\frac{4-N^2}{4N}$	0	0	0	0	$-\frac{N}{4}$	0

$$C_{gg \rightarrow ggg}^{35} = \quad (B.18)$$

$$\begin{pmatrix}
 -\frac{N}{2} & 0 \\
 0 & 0 & \frac{N^2}{-4+N^2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 1 & \frac{N}{4} & \frac{-4+N^2}{4N} & 0 & 0 & 0 & 0 & \frac{-3-N}{4(1+N)} & \frac{-3+N}{4(-1+N)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & \frac{N^2}{4(-4+N^2)} & \frac{N}{4} & 0 & 0 & -\frac{N^2}{2(-4+N^2)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & -\frac{N}{4} & \frac{N}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & \frac{N}{4} & -\frac{N}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & -1 & 0 & 0 & -\frac{N}{2} & 0 & \frac{N(3+N)}{4(2+N)} & \frac{-(3-N)N}{4(-2+N)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{N}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & -1 & 0 & 0 & 0 & \frac{-2-N+N^2}{2N} & 0 & \frac{1}{2}(-1-N) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 1 & 0 & 0 & 0 & \frac{2-N-N^2}{2N} & 0 & 0 & \frac{1-N}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & \frac{-3-N}{4(2+N)} & \frac{3-N}{4(-2+N)} & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{N^2}{2(-4+N^2)} & 0 & \frac{N^2}{2(-4+N^2)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -\frac{N}{4} & 0 & -\frac{(-2+N)(3+N)}{8(2+N)} & \frac{(-3+N)(2+N)}{8(-2+N)} & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{N}{4} & -\frac{N}{8} & -\frac{N}{8} & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & \frac{2-N}{4} & \frac{6+N-N^2}{4N} & \frac{1}{4}(-2-N) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & \frac{2+N}{4} & \frac{6-N-N^2}{4N} & 0 & \frac{2-N}{4} & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & -\frac{1-N}{2N} & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{2-N}{4} & \frac{2-N}{4} & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & \frac{-4-N^2}{4N} & -\frac{N}{4} & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & \frac{1-N}{2N} \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4}(-2-N) & \frac{2-N}{4} \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & \frac{4-N^2}{4N} & -\frac{N}{4}
 \end{pmatrix}$$

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$$C_{gg \rightarrow ggg}^{45} =$$

(B.19)

$$\begin{pmatrix} -\frac{N}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{N^2}{-1+4N^2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & \frac{N}{4} & \frac{4-N^2}{4N} & 0 & 0 & 0 & 0 & \frac{3-N}{4(1+N)} & \frac{3-N}{4(-1+N)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{N^2}{4(-4+N^2)} & -\frac{N}{4} & 0 & 0 & \frac{N^2}{2(-4+N^2)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{N}{4} & -\frac{N}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{N}{4} & -\frac{N}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -\frac{N}{2} & 0 & -\frac{N(3+N)}{4(2+N)} & \frac{(-3+N)N}{4(-2+N)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{N}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & \frac{2+N \cdot N^2}{2N} & 0 & \frac{1}{2}(-1-N) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & \frac{-2+N \cdot N^2}{2N} & 0 & \frac{1-N}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{3-N}{4(2+N)} & \frac{-3-N}{4(-2+N)} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{N^2}{2(-4+N^2)} & 0 & -\frac{N^2}{2(-4+N^2)} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & \frac{N}{4} & 0 & \frac{(-2+N)(3+N)}{8(2+N)} & \frac{(-3+N)(2+N)}{8(-2+N)} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{N}{4} & \frac{N}{8} & \frac{N}{8} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & \frac{1}{4}(-2+N) & \frac{(-3+N)(2+N)}{4N} & \frac{1}{4}(-2-N) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & \frac{1}{4}(-2-N) & \frac{(-2+N)(3+N)}{4N} & 0 & \frac{2-N}{4} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1+N}{2N} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{2-N}{4} & \frac{1}{4}(-2-N) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & \frac{4-N^2}{4N} & -\frac{N}{4} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4}(-2-N) & \frac{1}{4}(-2+N) \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ & & & & & & & & & & & & & & & & & \frac{-4+N^2}{4N} & -\frac{N}{4} \end{pmatrix}$$

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$$C_{gg \rightarrow ggg}^{13} =$$

0	$\frac{N}{-1+N^2}$	$\frac{N^2}{2(-1+N^2)}$	$\frac{-4+N^2}{2(-1+N^2)}$	0	0	0	0	$\frac{N^2(3+N)}{4(-1+N)(1+N)^2}$	$\frac{(-3+N)N^2}{4(-1+N)^2(1+N)}$	0	0
$\frac{N}{-1+N^2}$	0	$-\frac{N^2}{2(-1+N^2)}$	0	$\frac{-4+N^2}{2(-1+N^2)}$	0	0	0	0	0	0	0
$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{N}{8}$	$\frac{4-N^2}{8N}$	$\frac{-4+N^2}{8N}$	$\frac{-4+N^2}{8N}$	0	0	$\frac{3+N}{8(1+N)}$	$\frac{3-N}{8(-1+N)}$	0	0
$\frac{N^2}{2(-4+N^2)}$	0	$-\frac{N^3}{8(-4+N^2)}$	$-\frac{N}{8}$	$\frac{N(-12+N^2)}{8(-4+N^2)}$	$\frac{N}{8}$	$\frac{N^2}{4(-4+N^2)}$	$\frac{N}{2(-4+N^2)}$	0	0	0	0
0	$\frac{N^2}{2(-4+N^2)}$	$\frac{N^3}{8(-4+N^2)}$	$\frac{N(-12+N^2)}{8(-4+N^2)}$	$-\frac{N}{8}$	$-\frac{N}{8}$	0	0	$\frac{N^3(3+N)}{8(2+N)^2(-2+N+N^2)}$	$-\frac{(-3+N)N^3}{8(-2+N)^2(-2+N+N^2)}$	$-\frac{N^2}{4(-4+N^2)}$	$\frac{N}{2(-4+N^2)}$
0	0	$\frac{N^3}{8(-4+N^2)}$	$\frac{N}{8}$	$-\frac{N}{8}$	$-\frac{N}{8}$	$\frac{N^2}{4(-4+N^2)}$	0	0	0	$\frac{N^2}{4(-4+N^2)}$	0
0	0	0	$\frac{1}{2}$	0	$\frac{1}{2}$	$-\frac{N}{4}$	$-\frac{1}{4}$	$-\frac{N(3+N)}{8(2+N)}$	$\frac{(-3+N)N}{8(-2+N)}$	0	0
0	0	0	$-\frac{N}{-4+N^2}$	0	0	$-\frac{N^2}{4(-4+N^2)}$	$-\frac{N}{4}$	$\frac{N^3(3+N)}{8(-2+N)(2+N)^2}$	$\frac{(-3+N)N^3}{8(-2+N)^2(2+N)}$	0	$\frac{N}{2(-4+N^2)}$
1	0	$\frac{1}{2}$	0	$\frac{-2+N}{2N}$	0	$\frac{2+N-N^2}{4N}$	$\frac{-2-N+N^2}{4N}$	$\frac{1}{4}(-1-N)$	0	0	0
1	0	$-\frac{1}{2}$	0	$\frac{-2-N}{2N}$	0	$\frac{-2+N+N^2}{4N}$	$\frac{-2+N+N^2}{4N}$	0	$\frac{1-N}{4}$	0	0
0	0	0	0	$-\frac{1}{2}$	$\frac{1}{2}$	0	0	0	0	$-\frac{N}{4}$	$\frac{1}{4}$
0	0	0	0	$\frac{N}{-4+N^2}$	0	0	$\frac{N}{2(-4+N^2)}$	0	0	$\frac{N^2}{4(-4+N^2)}$	$-\frac{N}{4}$
0	0	0	0	0	1	0	$\frac{1}{2}$	0	0	0	$-\frac{1}{2}$
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	-1	0	0	$-\frac{1}{2}$	0	0	$\frac{1}{2}$	0
0	0	0	0	1	0	0	$\frac{1}{2}$	0	0	$\frac{1}{2}$	0
0	1	$-\frac{1}{2}$	$\frac{-2+N}{2N}$	0	0	0	0	$\frac{2+N-N^2}{4N(2+3N+N^2)}$	$\frac{-3+N}{4(-1+N)N}$	$\frac{-2-N+N^2}{4N}$	$\frac{-2-N+N^2}{4N}$
0	0	0	-1	0	0	$-\frac{1}{2}$	0	$-\frac{1}{2(2+N)}$	$\frac{3-N}{2(-2+N)}$	0	$-\frac{1}{2}$
0	0	-1	0	0	0	$\frac{-2+N}{2N}$	0	$-\frac{1}{2}$	0	$\frac{-2+N}{2N}$	0
0	1	$\frac{1}{2}$	$\frac{-2-N}{2N}$	0	0	0	0	$\frac{3+N}{4N(1+N)}$	$\frac{2-N+N^2}{4N(2-3N+N^2)}$	$\frac{2-N+N^2}{4N}$	$\frac{-2+N+N^2}{4N}$
0	0	0	1	0	0	$-\frac{1}{2}$	0	$\frac{3+N}{2(2+N)}$	$-\frac{1}{2(-2+N)}$	0	$\frac{1}{2}$
0	0	-1	0	0	0	$\frac{-2-N}{2N}$	0	0	$-\frac{1}{2}$	$\frac{-2-N}{2N}$	0

$$C_{gg \rightarrow ggg}^{14} =$$

0	$-\frac{N}{-1+N^2}$	$\frac{N^2}{2(-1+N^2)}$	$\frac{4-N^2}{2(-1+N^2)}$	0	0	0	0	$-\frac{N^2(3+N)}{4(-1+N)(1+N)^2}$	$-\frac{(-3+N)N^2}{4(-1+N)^2(1+N)}$	0	0
$-\frac{N}{-1+N^2}$	0	$\frac{N^2}{2(-1+N^2)}$	0	$\frac{4-N^2}{2(-1+N^2)}$	0	0	0	0	0	0	0
$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{N}{8}$	$\frac{-4+N^2}{8N}$	$\frac{-4+N^2}{8N}$	$\frac{4-N^2}{8N}$	0	0	$\frac{-3-N}{8(1+N)}$	$\frac{-3+N}{8(-1+N)}$	0	0
$-\frac{N^2}{2(-4+N^2)}$	0	$\frac{N^3}{8(-4+N^2)}$	$-\frac{N}{8}$	$-\frac{N(-12+N^2)}{8(-4+N^2)}$	$\frac{N}{8}$	$-\frac{N^2}{4(-4+N^2)}$	$-\frac{N}{2(-4+N^2)}$	0	0	0	0
0	$-\frac{N^2}{2(-4+N^2)}$	$\frac{N^3}{8(-4+N^2)}$	$-\frac{N(-12+N^2)}{8(-4+N^2)}$	$-\frac{N}{8}$	$\frac{N}{8}$	0	0	$-\frac{N^3(3+N)}{8(2+N)^2(-2+N+N^2)}$	$\frac{(-3+N)N^3}{8(-2+N)^2(-2+N+N^2)}$	$-\frac{N^2}{4(-4+N^2)}$	$-\frac{N}{2(-4+N^2)}$
0	0	$-\frac{N^3}{8(-4+N^2)}$	$\frac{N}{8}$	$\frac{N}{8}$	$-\frac{N}{8}$	$-\frac{N^2}{4(-4+N^2)}$	0	0	0	$-\frac{N^2}{4(-4+N^2)}$	0
0	0	0	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{N}{4}$	$-\frac{1}{4}$	$\frac{N(3+N)}{8(2+N)}$	$-\frac{(-3+N)N}{8(-2+N)}$	0	0
0	0	0	$-\frac{N}{-4+N^2}$	0	0	$-\frac{N^2}{4(-4+N^2)}$	$-\frac{N}{4}$	$-\frac{N^3(3+N)}{8(-2+N)(2+N)^2}$	$-\frac{(-3+N)N^3}{8(-2+N)^2(2+N)}$	0	$-\frac{N}{2(-4+N^2)}$
-1	0	$-\frac{1}{2}$	0	$\frac{2-N}{2N}$	0	$\frac{-2-N+N^2}{4N}$	$\frac{2+N-N^2}{4N}$	$\frac{1}{4}(-1-N)$	0	0	0
-1	0	$\frac{1}{2}$	0	$\frac{2+N}{2N}$	0	$\frac{2-N-N^2}{4N}$	$\frac{2-N-N^2}{4N}$	0	$\frac{1-N}{4}$	0	0
0	0	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	0	$-\frac{N}{4}$	$-\frac{1}{4}$
0	0	0	0	$-\frac{N}{-4+N^2}$	0	0	$-\frac{N}{2(-4+N^2)}$	0	0	$-\frac{N^2}{4(-4+N^2)}$	$-\frac{N}{4}$
0	0	0	0	0	-1	0	$\frac{1}{2}$	0	0	0	$\frac{1}{2}$
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	$\frac{1}{2}$	0	0	$-\frac{1}{2}$	0
0	0	0	0	-1	0	0	$-\frac{1}{2}$	0	0	$-\frac{1}{2}$	0
0	-1	$-\frac{1}{2}$	$\frac{2-N}{2N}$	0	0	0	0	$\frac{-2-N+N^2}{4N(2+3N+N^2)}$	$\frac{3-N}{4(-1+N)N}$	$\frac{-2-N+N^2}{4N}$	$\frac{2+N-N^2}{4N}$
0	0	0	1	0	0	$-\frac{1}{2}$	0	$\frac{1}{2(2+N)}$	$\frac{-3+N}{2(-2+N)}$	0	$\frac{1}{2}$
0	0	1	0	0	0	$\frac{2-N}{2N}$	0	$-\frac{1}{2}$	0	$\frac{2-N}{2N}$	0
0	-1	$\frac{1}{2}$	$\frac{2+N}{2N}$	0	0	0	0	$\frac{-3-N}{4N(1+N)}$	$\frac{-2+N-N^2}{4N(2-3N+N^2)}$	$\frac{2-N+N^2}{4N}$	$\frac{2-N+N^2}{4N}$
0	0	0	-1	0	0	$-\frac{1}{2}$	0	$\frac{-3-N}{2(2+N)}$	$\frac{1}{2(-2+N)}$	0	$-\frac{1}{2}$
0	0	1	0	0	0	$\frac{2+N}{2N}$	0	0	$-\frac{1}{2}$	$\frac{2+N}{2N}$	0

0	0	0	0	0	0	0	0	0	0
0	0	0	$\frac{N^2(3+N)}{4(-1+N)(1+N)^2}$	0	0	$\frac{(-3+N)N^2}{4(-1+N)^2(1+N)}$	0	0	0
0	0	0	$\frac{-3-N}{8(1+N)}$	0	$\frac{-3-N}{8N}$	$\frac{-3-N}{8(1+N)}$	0	$\frac{3-N}{8N}$	0
0	0	0	$\frac{N^3(3+N)}{8(2+N)^2(-2-N+N^2)}$	$-\frac{N^2(3+N)}{8(-2+N)(2+N)^2}$	0	$-\frac{(-3+N)N^3}{8(-2+N)^2(-2+N+N^2)}$	$\frac{(-3+N)N^2}{8(-2+N)^2(2+N)}$	0	0
0	$-\frac{N^2(3+N)}{8(-2+N)(2+N)^2}$	$\frac{(-3+N)N^2}{8(-2+N)^2(2+N)}$	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	$\frac{-3-N}{8(2+N)}$	$\frac{3+N}{8(2+N)}$	0	$\frac{3-N}{8(-2+N)}$	$\frac{3-N}{8(-2+N)}$
0	$-\frac{N^2(3+N)}{8(-2+N)(2+N)^2}$	$\frac{(-3+N)N^2}{8(-2+N)^2(2+N)}$	0	0	0	0	0	0	0
0	0	0	$\frac{2+N+N^2}{4N(2+3N+N^2)}$	$\frac{2+N-N^2}{4N^2(2+N)}$	$\frac{-1-N}{4N}$	$\frac{-3+N}{4(-1+N)N}$	$\frac{-3-2N+N^2}{4N^2}$	0	0
0	0	0	$\frac{3+N}{4N(1+N)}$	$\frac{3-2N-N^2}{4N^2}$	0	$\frac{2-N-N^2}{4N(2-3N+N^2)}$	$\frac{2-N-N^2}{4(-2+N)N^2}$	$\frac{1-N}{4N}$	0
0	$\frac{3+N}{8(2+N)}$	$\frac{-3+N}{8(-2+N)}$	$\frac{N(3+N)}{8(2+N)}$	0	$\frac{3+N}{8(2+N)}$	$-\frac{(-3+N)N}{8(-2+N)}$	0	$\frac{3-N}{8(-2+N)}$	$\frac{3-N}{8(-2+N)}$
0	0	0	$\frac{N^3(3+N)}{8(-2+N)(2+N)^2}$	$-\frac{N^2(3+N)}{8(-2+N)(2+N)^2}$	0	$\frac{(-3+N)N^3}{8(-2+N)^2(2+N)}$	$\frac{(-3+N)N^2}{8(-2+N)^2(2+N)}$	0	0
0	$\frac{(-2+N)(3+N)}{16(2+N)}$	$-\frac{(-3+N)(2+N)}{16(-2+N)}$	0	$-\frac{(-2+N)(3+N)}{16(2+N)}$	$-\frac{-6-11N-3N^2}{16(2+N)}$	0	$\frac{(-3+N)(2+N)}{16(-2+N)}$	$\frac{-6+11N-3N^2}{16(-2+N)}$	0
$-\frac{3N}{8}$	$\frac{N}{16}$	$\frac{N}{16}$	0	$-\frac{N}{16}$	$\frac{N}{16}$	0	$-\frac{N}{16}$	$-\frac{N}{16}$	0
$-\frac{6-N+N^2}{8N}$	$\frac{1}{8}(-2-3N)$	0	$-\frac{1}{2(2+N)}$	$-\frac{4-4N+2N^2+N^3}{8N(2+N)}$	$\frac{2+N}{8}$	$\frac{3-N}{2(-2+N)}$	$\frac{-6+2N-3N^2-N^3}{4(-2+N)N}$	0	0
$\frac{(-2+N)(3+N)}{8N}$	0	$\frac{1}{8}(2-3N)$	$\frac{3+N}{2(2+N)}$	$\frac{6+2N-3N^2-N^3}{4N(2+N)}$	0	$-\frac{1}{2(-2+N)}$	$\frac{4-4N-2N^2+N^3}{8(-2+N)N}$	$\frac{2-N}{8}$	0
0	$\frac{2+N-N^2}{4N^2(2+N)}$	$\frac{-3-2N+N^2}{4N^2}$	$\frac{1}{4}(-1-N)$	0	$\frac{1+N}{4N}$	0	0	0	0
$-\frac{(-3+N)(2+N)}{8N}$	$\frac{-4-4N+2N^2+N^3}{8N(2+N)}$	$\frac{-6+2N+3N^2-N^3}{4(-2+N)N}$	0	$\frac{1}{8}(-2-3N)$	$\frac{1}{8}(-2-N)$	0	0	0	0
$\frac{12-4N-3N^2+N^3}{8N^2}$	$\frac{-4+N^2}{8N}$	0	$\frac{1}{2}$	$\frac{4-N^2}{8N}$	$\frac{1}{8}(-4-3N)$	0	0	0	0
0	$\frac{3-2N-N^2}{4N^2}$	$\frac{2-N-N^2}{4(-2+N)N^2}$	0	0	0	$\frac{1-N}{4}$	0	$\frac{-1+N}{4N}$	0
$-\frac{(-2+N)(3+N)}{8N}$	$\frac{6+2N-3N^2-N^3}{4N(2+N)}$	$\frac{4-4N-2N^2+N^3}{8(-2+N)N}$	0	0	0	0	$\frac{1}{8}(2-3N)$	$\frac{1}{8}(-2+N)$	0
$\frac{12+4N-3N^2-N^3}{8N^2}$	0	$\frac{4-N^2}{8N}$	0	0	0	$\frac{1}{2}$	$\frac{4+N^2}{8N}$	$\frac{1}{8}(4-3N)$	0

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	$-\frac{N^2(3+N)}{4(-1+N)(1+N)^2}$	0	0	$-\frac{(-3+N)N^2}{4(-1+N)^2(1+N)}$	0
0	0	0	0	0	$\frac{-3-N}{8(1+N)}$	0	$\frac{3+N}{8N}$	$\frac{-3-N}{8(1+N)}$	$\frac{-3-N}{8N}$
0	0	0	0	0	$-\frac{N^3(3+N)}{8(2+N)^2(-2-N+N^2)}$	$\frac{N^2(3+N)}{8(-2+N)(2+N)^2}$	0	$\frac{(-3+N)N^3}{8(-2+N)^2(-2+N+N^2)}$	$-\frac{(-3+N)N^2}{8(-2+N)^2(2+N)}$
0	0	$\frac{N^2(3+N)}{8(-2+N)(2+N)^2}$	$-\frac{(-3+N)N^2}{8(-2+N)^2(2+N)}$	0	0	0	0	0	0
$-\frac{N^2}{4(-4+N^2)}$	0	0	0	0	0	0	0	0	0
0	0	0	0	0	$\frac{-3-N}{8(2+N)}$	$\frac{-3-N}{8(2+N)}$	0	$\frac{3-N}{8(-2+N)}$	$\frac{-3+N}{8(-2+N)}$
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	$\frac{N^2(3+N)}{8(-2+N)(2+N)^2}$	$-\frac{(-3+N)N^2}{8(-2+N)^2(2+N)}$	0	0	0
0	0	0	0	0	$\frac{-2-N-N^2}{4N(2+3N+N^2)}$	$\frac{-2-N-N^2}{4N^2(2+N)}$	$\frac{-1-N}{4N}$	$\frac{3-2N-N^2}{4N^2}$	0
0	0	0	0	0	$\frac{-3-N}{4N(1+N)}$	$\frac{-3+2N+N^2}{4N^2}$	0	$\frac{-2-N-N^2}{4N(2-3N+N^2)}$	$\frac{1-N}{4N}$
0	0	0	0	0	$\frac{-3-N}{8(2+N)}$	$\frac{3-N}{8(2+N)}$	$-\frac{(-3+N)N}{8(-2+N)}$	0	$\frac{-3+N}{8(-2+N)}$
$\frac{N^2}{4(-4+N^2)}$	0	0	0	$-\frac{N^3(3+N)}{8(-2+N)(2+N)^2}$	$\frac{N^2(3+N)}{8(-2+N)(2+N)^2}$	0	$-\frac{(-3+N)N^3}{8(-2+N)^2(2+N)}$	$-\frac{(-3+N)N^2}{8(-2+N)^2(2+N)}$	0
$-\frac{3N}{8}$	0	$-\frac{(-2+N)(3+N)}{16(2+N)}$	$\frac{(-3+N)(2+N)}{16(-2+N)}$	0	$-\frac{(-2+N)(3+N)}{16(2+N)}$	$\frac{6+11N+3N^2}{16(2+N)}$	0	$\frac{(-3+N)(2+N)}{16(-2+N)}$	$\frac{6-11N+3N^2}{16(-2+N)}$
0	$-\frac{3N}{8}$	$-\frac{N}{16}$	$-\frac{N}{16}$	0	$-\frac{N}{16}$	$-\frac{N}{16}$	0	$-\frac{N}{16}$	$\frac{N}{16}$
$\frac{2-N}{8}$	$-\frac{(-3+N)(2+N)}{8N}$	$\frac{1}{8}(-2-3N)$	0	$\frac{1}{2(2+N)}$	$\frac{4+4N-2N^2-N^3}{8N(2+N)}$	$\frac{2+N}{8}$	$\frac{-3+N}{2(-2+N)}$	$\frac{-6+2N-3N^2+N^3}{4(-2+N)N}$	0
$\frac{2+N}{8}$	$-\frac{(-2+N)(3+N)}{8N}$	0	$\frac{1}{8}(2-3N)$	$\frac{-3-N}{2(2+N)}$	$\frac{-6+2N-3N^2-N^3}{4N(2+N)}$	0	$\frac{1}{2(-2+N)}$	$\frac{-4+4N-2N^2-N^3}{8(-2+N)N}$	$\frac{2-N}{8}$
0	0	$\frac{-2-N+N^2}{4N^2(2+N)}$	$\frac{3+2N-N^2}{4N^2}$	$\frac{1}{4}(-1-N)$	0	$\frac{-1-N}{4N}$	0	0	0
$\frac{2-N}{8}$	$-\frac{(-3+N)(2+N)}{8N}$	$\frac{4+4N-2N^2+N^3}{8N(2+N)}$	$\frac{6-2N-3N^2+N^3}{4(-2+N)N}$	0	$\frac{1}{8}(-2-3N)$	$\frac{2-N}{8}$	0	0	0
$\frac{-4-4N+3N^2}{8N}$	$-\frac{(2+N)(6-5N+N^2)}{8N^2}$	$\frac{-4+N^2}{8N}$	0	$-\frac{1}{2}$	$\frac{-4-N^2}{8N}$	$\frac{1}{8}(-4-3N)$	0	0	0
0	0	$\frac{-3+2N+N^2}{4N^2}$	$\frac{-2+N-N^2}{4(-2+N)N^2}$	0	0	0	$\frac{1-N}{4}$	$\frac{1-N}{4N}$	0
$\frac{2+N}{8}$	$-\frac{(-2+N)(3+N)}{8N}$	$\frac{-6-2N+3N^2+N^3}{4N(2+N)}$	$\frac{-4+4N+2N^2+N^3}{8(-2+N)N}$	0	0	0	0	$\frac{1}{8}(2-3N)$	$\frac{2-N}{8}$
$\frac{-4+4N+3N^2}{8N}$	$\frac{(-2+N)(6+5N+N^2)}{8N^2}$	0	$\frac{4-N^2}{8N}$	0	0	0	$-\frac{1}{2}$	$\frac{4-N^2}{8N}$	$\frac{1}{8}(4-3N)$

$$C_{gg \rightarrow ggg}^{23} =$$

0	$-\frac{N}{-1+N^2}$	$-\frac{N^2}{2(-1+N^2)}$	$\frac{4-N^2}{2(-1+N^2)}$	0	0	0	0	$-\frac{N^2(3+N)}{4(-1+N)(1+N)^2}$	$-\frac{(-3+N)N^2}{4(-1+N)^2(1+N)}$	0	0	0
$-\frac{N}{-1+N^2}$	0	$-\frac{N^2}{2(-1+N^2)}$	0	$\frac{4-N^2}{2(-1+N^2)}$	0	0	0	0	0	0	0	0
$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{N}{8}$	$\frac{4-N^2}{8N}$	$\frac{4-N^2}{8N}$	$\frac{4-N^2}{8N}$	0	0	$\frac{3+N}{8(1+N)}$	$\frac{3-N}{8(-1+N)}$	0	0	0
$-\frac{N^2}{2(-4+N^2)}$	0	$-\frac{N^3}{8(-4+N^2)}$	$-\frac{N}{8}$	$-\frac{N(-12+N^2)}{8(-4+N^2)}$	$-\frac{N}{8}$	$-\frac{N^2}{4(-4+N^2)}$	$-\frac{N}{2(-4+N^2)}$	0	0	0	0	0
0	$-\frac{N^2}{2(-4+N^2)}$	$-\frac{N^3}{8(-4+N^2)}$	$-\frac{N(-12+N^2)}{8(-4+N^2)}$	$-\frac{N}{8}$	$-\frac{N}{8}$	0	0	$-\frac{N^3(3+N)}{8(2+N)^2(-2+N+N^2)}$	$-\frac{(-3+N)N^3}{8(-2+N)^2(-2+N+N^2)}$	$\frac{N^2}{4(-4+N^2)}$	$-\frac{N}{2(-4+N^2)}$	0
0	0	$-\frac{N^3}{8(-4+N^2)}$	$-\frac{N}{8}$	$-\frac{N}{8}$	$-\frac{N}{8}$	$-\frac{N^2}{4(-4+N^2)}$	0	0	0	$-\frac{N^2}{4(-4+N^2)}$	0	$-\frac{N^2}{4(-4+N^2)}$
0	0	0	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{N}{4}$	$\frac{1}{4}$	$-\frac{N(3+N)}{8(2+N)}$	$-\frac{(-3+N)N}{8(-2+N)}$	0	0	0
0	0	0	$-\frac{N}{-4+N^2}$	0	0	$\frac{N^2}{4(-4+N^2)}$	$-\frac{N}{4}$	$-\frac{N^3(3+N)}{8(-2+N)(2+N)^2}$	$-\frac{(-3+N)N^3}{8(-2+N)^2(2+N)}$	0	$-\frac{N}{2(-4+N^2)}$	$-\frac{N^2}{4(-4+N^2)}$
-1	0	$\frac{1}{2}$	0	$\frac{2-N}{2N}$	0	$\frac{2+N-N^2}{4N}$	$\frac{2+N-N^2}{4N}$	$\frac{1}{4}(-1-N)$	0	0	0	0
-1	0	$-\frac{1}{2}$	0	$\frac{2+N}{2N}$	0	$\frac{-2+N-N^2}{4N}$	$\frac{2-N-N^2}{4N}$	0	$\frac{1-N}{4}$	0	0	0
0	0	0	0	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	0	$-\frac{N}{4}$	$\frac{1}{4}$	0
0	0	0	0	$-\frac{N}{-4+N^2}$	0	0	$-\frac{N}{-4+N^2}$	0	0	$\frac{N^2}{4(-4+N^2)}$	$-\frac{N}{4}$	$-\frac{N^2}{4(-4+N^2)}$
0	0	0	0	0	0	-1	$-\frac{1}{2}$	0	0	0	$-\frac{1}{2}$	$-\frac{3N}{8}$
0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	$\frac{1}{2}$	0	$\frac{1}{8}(-2+N)$
0	0	0	0	0	-1	0	0	$-\frac{1}{2}$	0	$\frac{1}{2}$	0	$\frac{1}{8}(-2-N)$
0	-1	$\frac{1}{2}$	$\frac{2-N}{2N}$	0	0	0	0	$\frac{-2-N-N^2}{4N(2+3N+N^2)}$	$\frac{3-N}{4(-1+N)N}$	$\frac{2+N-N^2}{4N}$	$\frac{2+N-N^2}{4N}$	0
0	0	0	1	0	0	$\frac{1}{2}$	0	$\frac{1}{2(2+N)}$	$\frac{-3+N}{2(-2+N)}$	0	$\frac{1}{2}$	$\frac{1}{8}(-2+N)$
0	0	1	0	0	0	$\frac{2-N}{2N}$	0	$\frac{1}{2}$	0	$\frac{2-N}{2N}$	0	$\frac{-4-4N+3N^2}{8N}$
0	-1	$-\frac{1}{2}$	$\frac{2+N}{2N}$	0	0	0	0	$\frac{-3-N}{4N(1+N)}$	$\frac{-2+N-N^2}{4N(2-3N+N^2)}$	$\frac{-2+N+N^2}{4N}$	$\frac{2-N-N^2}{4N}$	0
0	0	0	-1	0	0	$\frac{1}{2}$	0	$\frac{-3-N}{2(2+N)}$	$\frac{1}{2(-2+N)}$	0	$-\frac{1}{2}$	$\frac{1}{8}(-2-N)$
0	0	1	0	0	0	$\frac{2+N}{2N}$	0	0	$\frac{1}{2}$	$\frac{2+N}{2N}$	0	$\frac{-4+4N+3N^2}{8N}$

$$C_{gg \rightarrow ggg}^{24} =$$

0	$\frac{N}{-1+N^2}$	$-\frac{N^2}{2(-1+N^2)}$	$\frac{-4+N^2}{2(-1+N^2)}$	0	0	0	0	$\frac{N^2(3+N)}{4(-1+N)(1+N)^2}$	$\frac{(-3+N)N^2}{4(-1+N)^2(1+N)}$	0	0	0
$\frac{N}{-1+N^2}$	0	$\frac{N^2}{2(-1+N^2)}$	0	$\frac{-4+N^2}{2(-1+N^2)}$	0	0	0	0	0	0	0	0
$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{N}{8}$	$\frac{-4+N^2}{8N}$	$\frac{4-N^2}{8N}$	$\frac{-4+N^2}{8N}$	0	0	$\frac{-3-N}{8(1+N)}$	$\frac{-3+N}{8(-1+N)}$	0	0	0
$\frac{N^2}{2(-4+N^2)}$	0	$\frac{N^3}{8(-4+N^2)}$	$-\frac{N}{8}$	$\frac{N(-12+N^2)}{8(-4+N^2)}$	$-\frac{N}{8}$	$-\frac{N^2}{4(-4+N^2)}$	$\frac{N}{2(-4+N^2)}$	0	0	0	0	0
0	$\frac{N^2}{2(-4+N^2)}$	$-\frac{N^3}{8(-4+N^2)}$	$\frac{N(-12+N^2)}{8(-4+N^2)}$	$-\frac{N}{8}$	$\frac{N}{8}$	0	0	$\frac{N^3(3+N)}{8(2+N)^2(-2+N+N^2)}$	$-\frac{(-3+N)N^3}{8(-2+N)^2(-2+N+N^2)}$	$\frac{N^2}{4(-4+N^2)}$	$\frac{N}{2(-4+N^2)}$	0
0	0	$\frac{N^3}{8(-4+N^2)}$	$-\frac{N}{8}$	$\frac{N}{8}$	$-\frac{N}{8}$	$\frac{N^2}{4(-4+N^2)}$	0	0	0	$\frac{N^2}{4(-4+N^2)}$	0	0
0	0	0	$-\frac{1}{2}$	0	$\frac{1}{2}$	$-\frac{N}{4}$	$\frac{1}{4}$	$\frac{N(3+N)}{8(2+N)}$	$-\frac{(-3+N)N}{8(-2+N)}$	0	0	0
0	0	0	$\frac{N}{-4+N^2}$	0	0	$\frac{N^2}{4(-4+N^2)}$	$-\frac{N}{4}$	$\frac{N^3(3+N)}{8(-2+N)(2+N)^2}$	$\frac{(-3+N)N^3}{8(-2+N)^2(2+N)}$	0	$\frac{N}{2(-4+N^2)}$	0
1	0	$-\frac{1}{2}$	0	$\frac{-2+N}{2N}$	0	$\frac{-2+N-N^2}{4N}$	$\frac{-2+N-N^2}{4N}$	$\frac{1}{4}(-1-N)$	0	0	0	0
1	0	$\frac{1}{2}$	0	$\frac{-2+N}{2N}$	0	$\frac{2-N-N^2}{4N}$	$\frac{-2+N-N^2}{4N}$	0	$\frac{1-N}{4}$	0	0	0
0	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	0	$-\frac{N}{4}$	$-\frac{1}{4}$	0
0	0	0	0	$\frac{N}{-4+N^2}$	0	0	0	$\frac{N}{2(-4+N^2)}$	0	$-\frac{N^2}{4(-4+N^2)}$	$-\frac{N}{4}$	0
0	0	0	0	0	0	1	0	$-\frac{1}{2}$	0	0	$\frac{1}{2}$	0
0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	-1	0	0	0	$-\frac{1}{2}$	0	0	$-\frac{1}{2}$	0
0	0	0	0	0	0	0	0	$\frac{1}{2}$	0	0	$-\frac{1}{2}$	0
0	1	$\frac{1}{2}$	$\frac{-2+N}{2N}$	0	0	0	0	$\frac{2+N+N^2}{4N(2+3N+N^2)}$	$\frac{-3+N}{4(-1+N)N}$	$\frac{2+N-N^2}{4N}$	$\frac{-2+N-N^2}{4N}$	0
0	0	0	-1	0	0	$\frac{1}{2}$	0	$-\frac{1}{2(2+N)}$	$\frac{3-N}{2(-2+N)}$	0	$-\frac{1}{2}$	0
0	0	-1	0	0	0	$\frac{-2+N}{2N}$	0	$\frac{1}{2}$	0	$\frac{-2+N}{2N}$	0	0
0	1	$-\frac{1}{2}$	$\frac{-2+N}{2N}$	0	0	0	0	$\frac{3+N}{4N(1+N)}$	$\frac{2-N+N^2}{4N(2-3N+N^2)}$	$\frac{-2+N-N^2}{4N}$	$\frac{-2+N-N^2}{4N}$	0
0	0	0	1	0	0	$\frac{1}{2}$	0	$\frac{3+N}{2(2+N)}$	$-\frac{1}{2(-2+N)}$	0	$\frac{1}{2}$	0
0	0	-1	0	0	0	$\frac{-2+N}{2N}$	0	0	$\frac{1}{2}$	$\frac{-2+N}{2N}$	0	0



(B.22)

0	0	0	0	0	0	0	0	0	0
0	0	0	0	$-\frac{N^2(3+N)}{4(-1+N)(1+N)^2}$	0	0	$-\frac{(-3+N)N^2}{4(-1+N)^2(1+N)}$	0	0
0	0	0	0	$\frac{3+N}{8(1+N)}$	0	$\frac{3+N}{8N}$	$\frac{3-N}{8(-1+N)}$	0	$-\frac{3+N}{8N}$
0	0	0	0	$-\frac{N^3(3+N)}{8(2+N)^2(-2+N+N^2)}$	$\frac{N^2(3+N)}{8(-2+N)(2+N)^2}$	0	$\frac{(-3+N)N^3}{8(-2+N)^2(-2+N+N^2)}$	$-\frac{(-3+N)N^2}{8(-2+N)^2(2+N)}$	0
0	0	$\frac{N^2(3+N)}{8(-2+N)(2+N)^2}$	$-\frac{(-3+N)N^2}{8(-2+N)^2(2+N)}$	0	0	0	0	0	0
$-\frac{N^2}{4(-4+N^2)}$	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
$-\frac{N^2}{4(-4+N^2)}$	0	$\frac{N^2(3+N)}{8(-2+N)(2+N)^2}$	$-\frac{(-3+N)N^2}{8(-2+N)^2(2+N)}$	0	0	0	0	0	0
0	0	0	0	$\frac{-2-N-N^2}{4N(2+3N+N^2)}$	$\frac{-2-N-N^2}{4N^2(2+N)}$	$\frac{1+N}{4N}$	$\frac{3-N}{4(-1+N)N}$	$\frac{3+2N-N^2}{4N^2}$	0
0	0	0	0	$\frac{-3-N}{4N(1+N)}$	$\frac{-3+2N+N^2}{4N^2}$	0	$\frac{-2-N-N^2}{4N(2-3N+N^2)}$	$\frac{-2-N-N^2}{4(-2+N)N^2}$	$-\frac{1+N}{4N}$
0	0	$\frac{3+N}{8(2+N)}$	$\frac{-3+N}{8(-2+N)}$	$-\frac{N(3+N)}{8(2+N)}$	0	$\frac{-3-N}{8(2+N)}$	$\frac{(-3+N)N}{8(-2+N)}$	0	$-\frac{-3+N}{8(-2+N)}$
$-\frac{N^2}{4(-4+N^2)}$	0	0	0	$-\frac{N^3(3+N)}{8(-2+N)(2+N)^2}$	$\frac{N^2(3+N)}{8(-2+N)(2+N)^2}$	0	$-\frac{(-3+N)N^3}{8(-2+N)^2(2+N)}$	$-\frac{(-3+N)N^2}{8(-2+N)^2(2+N)}$	0
$-\frac{3N}{8}$	0	$\frac{(-2+N)(3+N)}{16(2+N)}$	$-\frac{(-3+N)(2+N)}{16(-2+N)}$	0	$\frac{(-2+N)(3+N)}{16(2+N)}$	$\frac{6+11N+3N^2}{16(2+N)}$	0	$-\frac{(-3+N)(2+N)}{16(-2+N)}$	$\frac{6-11N+3N^2}{16(-2+N)}$
0	$-\frac{3N}{8}$	$\frac{N}{16}$	$\frac{N}{16}$	0	$\frac{N}{16}$	$-\frac{N}{16}$	0	$\frac{N}{16}$	$\frac{N}{16}$
$\frac{1}{8}(-2+N)$	$\frac{(-3+N)(2+N)}{8N}$	$\frac{1}{8}(-2-3N)$	0	$\frac{1}{2(2+N)}$	$\frac{4+4N-2N^2-N^3}{8N(2+N)}$	$\frac{1}{8}(-2-N)$	$\frac{-3+N}{2(-2+N)}$	$\frac{6-2N-3N^2-N^3}{4(-2+N)N}$	0
$\frac{1}{8}(-2-N)$	$\frac{(-2+N)(3+N)}{8N}$	0	$\frac{1}{8}(2-3N)$	$\frac{-3-N}{2(2+N)}$	$\frac{-6-2N+3N^2+N^3}{4N(2+N)}$	0	$\frac{1}{2(-2+N)}$	$\frac{-4+4N+2N^2-N^3}{8(-2+N)N}$	$\frac{1}{8}(-2+N)$
0	0	$\frac{-2-N-N^2}{4N^2(2+N)}$	$\frac{3+2N-N^2}{4N^2}$	$\frac{1}{4}(-1-N)$	0	$\frac{1+N}{4N}$	0	0	0
$\frac{1}{8}(-2+N)$	$\frac{(-3+N)(2+N)}{8N}$	$\frac{4+4N-2N^2-N^3}{8N(2+N)}$	$\frac{6-2N-3N^2-N^3}{4(-2+N)N}$	0	$\frac{1}{8}(-2-3N)$	$\frac{1}{8}(-2-N)$	0	0	0
$\frac{-4+4N+3N^2}{8N}$	$\frac{-1+2+4N+3N^2-N^3}{8N^2}$	$\frac{4-N^2}{8N}$	0	$\frac{1}{2}$	$\frac{4-N^2}{8N}$	$\frac{1}{8}(-4-3N)$	0	0	0
0	0	$\frac{-3+2N-N^2}{4N^2}$	$\frac{-2-N-N^2}{4(-2+N)N^2}$	0	0	0	$\frac{1-N}{4}$	0	$-\frac{1+N}{4N}$
$\frac{1}{8}(-2-N)$	$\frac{(-2+N)(3+N)}{8N}$	$\frac{-6-2N+3N^2+N^3}{4N(2+N)}$	$\frac{-4+4N+2N^2-N^3}{8(-2+N)N}$	0	0	0	0	$\frac{1}{8}(2-3N)$	$\frac{1}{8}(-2+N)$
$\frac{-4+4N+3N^2}{8N}$	$\frac{(-2+N)(6+5N+N^2)}{8N^2}$	0	$\frac{-4-N^2}{8N}$	0	0	0	$\frac{1}{2}$	$\frac{-4-N^2}{8N}$	$\frac{1}{8}(4-3N)$

(B.23)

0	0	0	0	0	0	0	0	0	0
0	0	0	0	$\frac{N^2(3+N)}{4(-1+N)(1+N)^2}$	0	0	$\frac{(-3+N)N^2}{4(-1+N)^2(1+N)}$	0	0
0	0	0	0	$\frac{3+N}{8(1+N)}$	0	$\frac{-3-N}{8N}$	$\frac{3-N}{8(-1+N)}$	0	$\frac{3-N}{8N}$
0	0	0	0	$\frac{N^3(3+N)}{8(2+N)^2(-2+N+N^2)}$	$-\frac{N^2(3+N)}{8(-2+N)(2+N)^2}$	0	$-\frac{(-3+N)N^3}{8(-2+N)^2(-2+N+N^2)}$	$\frac{(-3+N)N^2}{8(-2+N)^2(2+N)}$	0
0	0	$-\frac{N^2(3+N)}{8(-2+N)(2+N)^2}$	$\frac{(-3+N)N^2}{8(-2+N)^2(2+N)}$	0	0	0	0	0	0
$\frac{N^2}{4(-4+N^2)}$	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
$-\frac{N^2}{4(-4+N^2)}$	0	$-\frac{N^2(3+N)}{8(-2+N)(2+N)^2}$	$\frac{(-3+N)N^2}{8(-2+N)^2(2+N)}$	0	0	0	0	0	0
0	0	0	0	$\frac{2+N-N^2}{4N(2+3N+N^2)}$	$\frac{2+N-N^2}{4N^2(2+N)}$	$\frac{1+N}{4N}$	$\frac{-3+N}{4(-1+N)N}$	$\frac{-3-2N-N^2}{4N^2}$	0
0	0	0	0	$\frac{3+N}{4N(1+N)}$	$\frac{3-2N-N^2}{4N^2}$	0	$\frac{2-N-N^2}{4N(2-3N+N^2)}$	$\frac{2-N-N^2}{4(-2+N)N^2}$	$-\frac{1+N}{4N}$
0	0	$\frac{-3-N}{8(2+N)}$	$\frac{3-N}{8(-2+N)}$	$-\frac{N(3+N)}{8(2+N)}$	0	$\frac{3+N}{8(2+N)}$	$\frac{(-3+N)N}{8(-2+N)}$	0	$\frac{3-N}{8(-2+N)}$
$\frac{N^2}{4(-4+N^2)}$	0	0	0	$\frac{N^3(3+N)}{8(-2+N)(2+N)^2}$	$-\frac{N^2(3+N)}{8(-2+N)(2+N)^2}$	0	$\frac{(-3+N)N^3}{8(-2+N)^2(2+N)}$	$\frac{(-3+N)N^2}{8(-2+N)^2(2+N)}$	0
$-\frac{3N}{8}$	0	$-\frac{(-2+N)(3+N)}{16(2+N)}$	$\frac{(-3+N)(2+N)}{16(-2+N)}$	0	$\frac{(-2+N)(3+N)}{16(2+N)}$	$\frac{-6-11N-3N^2}{16(2+N)}$	0	$-\frac{(-3+N)(2+N)}{16(-2+N)}$	$\frac{-6+11N-3N^2}{16(-2+N)}$
0	$-\frac{3N}{8}$	$-\frac{N}{16}$	$-\frac{N}{16}$	0	$\frac{N}{16}$	$\frac{N}{16}$	0	$\frac{N}{16}$	$\frac{N}{16}$
$\frac{2-N}{8}$	$-\frac{(-3+N)(2+N)}{8N}$	$\frac{1}{8}(-2-3N)$	0	$-\frac{1}{2(2+N)}$	$\frac{-4-4N+2N^2+N^3}{8N(2+N)}$	$\frac{1}{8}(-2-N)$	$\frac{3-N}{2(-2+N)}$	$\frac{-6+2N+3N^2-N^3}{4(-2+N)N}$	0
$\frac{2+N}{8}$	$-\frac{(-2+N)(3+N)}{8N}$	0	$\frac{1}{8}(2-3N)$	$\frac{3+N}{2(2+N)}$	$\frac{6+2N+3N^2+N^3}{4N(2+N)}$	0	$-\frac{1}{2(-2+N)}$	$\frac{4-4N-2N^2-N^3}{8(-2+N)N}$	$\frac{1}{8}(-2+N)$
0	0	$\frac{2+N-N^2}{4N^2(2+N)}$	$\frac{-3-2N-N^2}{4N^2}$	$\frac{1}{4}(-1-N)$	0	$\frac{-1-N}{4N}$	0	0	0
$\frac{1}{8}(-2+N)$	$\frac{(-3+N)(2+N)}{8N}$	$\frac{-4+4N+2N^2+N^3}{8N(2+N)}$	$\frac{-6-2N+3N^2+N^3}{4(-2+N)N}$	0	$\frac{1}{8}(-2-3N)$	$\frac{2+N}{8}$	0	0	0
$\frac{4+4N-3N^2}{8N}$	$\frac{(2+N)(6+5N+N^2)}{8N^2}$	$\frac{4-N^2}{8N}$	0	$-\frac{1}{2}$	$\frac{-4+N^2}{8N}$	$\frac{1}{8}(-4-3N)$	0	0	0
0	0	$\frac{3+2N-N^2}{4N^2}$	$\frac{-2-N-N^2}{4(-2+N)N^2}$	0	0	0	$\frac{1-N}{4}$	0	$\frac{1-N}{4N}$
$\frac{1}{8}(-2-N)$	$\frac{(-2+N)(3+N)}{8N}$	$\frac{6+2N+3N^2-N^3}{4N(2+N)}$	$\frac{4-4N+2N^2+N^3}{8(-2+N)N}$	0	0	0	0	$\frac{1}{8}(2-3N)$	$\frac{2-N}{8}$
$\frac{4-4N-3N^2}{8N}$	$\frac{(-2+N)(6+5N+N^2)}{8N^2}$	0	$\frac{-4-N^2}{8N}$	0	0	0	$-\frac{1}{2}$	$\frac{4-N^2}{8N}$	$\frac{1}{8}(4-3N)$

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